A Theory of Land Finance and Investment-Led Growth

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Preliminary draft — comments very welcome

Abstract

Governments often heavily invest in infrastructure during early growth stages, while continuously expanding land supply to the private sector, as observed in the experiences of China, Singapore, and 19th-century US. To analyze these phenomena, I develop a growth model that incorporates both land and productive public capital. A benevolent government should optimally front-load public investment but maintain a *constant* land supply, which balances between fundraising through controlled supply and avoiding welfare loss from reduced land utilization. This optimal allocation can be implemented without necessitating government or private sector borrowing, or in a time-consistent manner. However, when a government is simultaneously discretionary and borrowing-constrained, a gradual increase in land supply becomes a reality, accounting for actual historical experiences. The mechanism is supported by preliminary empirical evidence from a panel of Chinese cities, showing that cities managed by officials nearing the end of their tenure and facing elevated borrowing costs tend to supply more land. Finally, I introduce a practical indexed land contract linking land supply to the provision of public goods. This contract can restore the optimal allocation, potentially serving as a template to improve real-world practices.

Keywords: neoclassical growth, public investment, optimal fiscal policy, financial constraints, time inconsistency, land contracts, Coase conjecture.

JEL: E62, G10, H27, H54, H63, L12, O23, O40, Q24

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1 Introduction

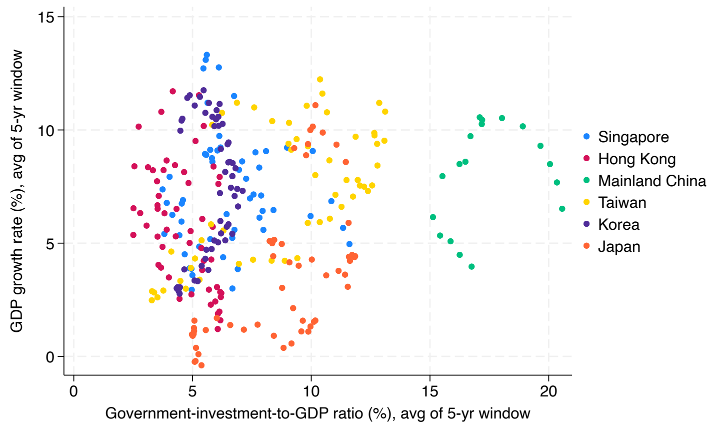
Throughout history, land has been one of the most important assets governments own. The management of land has long been a pressing concern, particularly for developing nations. Scholarly inquiry dates back to George (1879)'s celebrated proposal of land tax as the only tax a government needs. A century later, in an open letter to Gorbachev (Tideman et al., 1990), a group of distinguished economists including several Nobel laureates summarized people's intuitions to date and advocated for public land ownership. They suggested that land value rises from the provision of public goods (roads, utility networks, etc.), and thus is the most sensible revenue source for financing such public goods. They contended that western economies under-collect land rent and overuse taxes that impede their economies. Importantly, they cautioned against the outright sale of land, which may depress prices in the presence of financial constraints, and instead suggested public land ownership and annual rent collection by the government. This paper offers a theoretical framework that serves three purposes: i) a rigorous model to understand land that capture these ideas and *qualifies some of them*, ii) an explanation for governments' land management in practice, which qualitatively differs from the optimal policy, and iii) a proposal for a novel yet practical land contract that can improve social welfare.

Many governments have engaged in large-scale land transfers to the private sector to fund infrastructure investment, a practice known as "land finance." In the 19th century, the US federal and state governments utilized land sale revenue to fund massive "internal improvements" in roads, canals and railways (Goodrich, 1960; Feller, 1984). In recent decades, several Asian growth miracles, including Singapore, Hong Kong, and mainland China, derived about one-quarter of their government revenue from land transfers.[1] These Asian economies are also recognized for maintaining a high ratio of infrastructure investment to GDP during early periods of high growth (Figure 1). Looking into the future, financing public investment with land-based revenue is gaining traction across the developing world, and is indeed a focal point of various policy reports (Peterson, 2008; UK FCDO, 2015).[2] A better understanding of land finance, therefore, holds significant implications for policies that affect billions of people worldwide. Despite its significance, there exists a notable lack of theoretical analysis, which this paper aims to address.

[1] Figures A3 and A4 depict the ratio of land sale revenue to total government revenue for the aforementioned Asian economies in recent decades and for the US in the 19th century.

[2] Land finance has been implemented by places in India (Vyas, Vyas and Mishra, 2022), Vietnam (Nguyen et al., 2018), and Africa (Brown-Luthango, 2011; Berrisford, Cirolia and Palmer, 2018), among others.

Figure 1: Scatter plot of GDP growth rate against government-investment-to-GDP ratio



Notes: Data are from IMF Investment and Capital Stock Dataset (IMF, 2021). I plot all economies except mainland China from 1960 to 2019, and mainland China from 2000 (when economic growth and land finance took off).

Table 1: GDP growth rate strongly correlates with government-investment-to-GDP ratio

	(1) g^Y_{5y}	(2) g^Y_{5y}	(3) g^Y_{5y}	(4) g^Y_{5y}	(5) $(I/K)_{5y}$	(6) g^Y
$(I/K)_{5y}$	0.16	0.43	0.43			
	(0.05)	(0.08)	(0.08)			
year				-0.15	-0.05	
				(0.01)	(0.01)	
I/K						0.29
						(0.10)
Observations	293	293	275	293	298	315
R^2	0.03	0.30	0.29	0.74	0.75	0.13
Sample			excl. CHN			
Economy FE		Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses

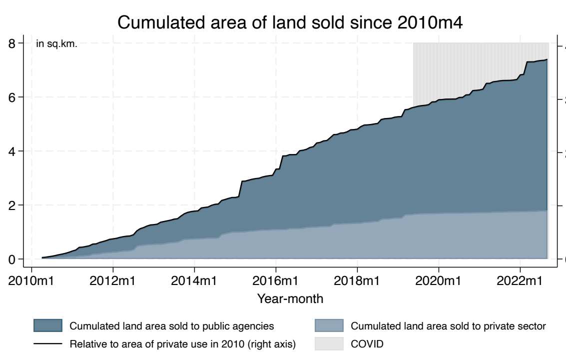
Notes: This analysis uses all observations in Figure 1. g^Y_{5y} is the 5-year average of GDP growth rate, and g^Y is the annual rate. $(I/K)_{5y}$ is the 5-year average of government-investment-to-GDP ratio, and I/K is the annual ratio. 5-year averages are used to filter out cyclical fluctuations (cols. 1-5), but the findings are robust to using annual variables too (col. 6). Column (3) excludes mainland China. Columns (2-6) include economy fixed effects.

We observe two common patterns across these episodes of land finance by different governments in different periods of time:

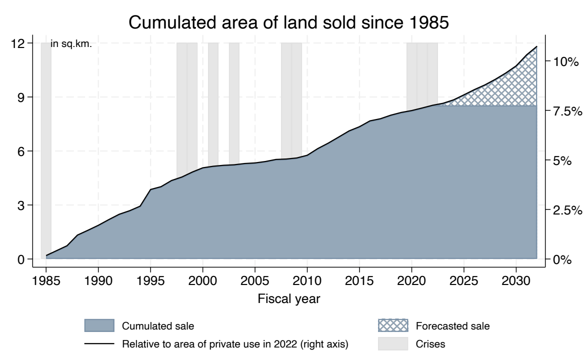
Stylized fact 1. (Early government investment) *The government-investment-to-GDP ratios are high in these economies during early fast-growing periods and decline as growth slows down.* Figure 1 and Table 1 show that government-investment-to-GDP ratio and GDP growth rate gradually decline over time for various recent growth miracles. Large-scale infrastructure investment, mostly through public spending, is widely viewed as a key development strategy. Indeed, developing countries tend to have higher public-investment-to-GDP ratios than developed countries (Gurara et al., 2018).

Figure 2: Added land supply since beginning of sample

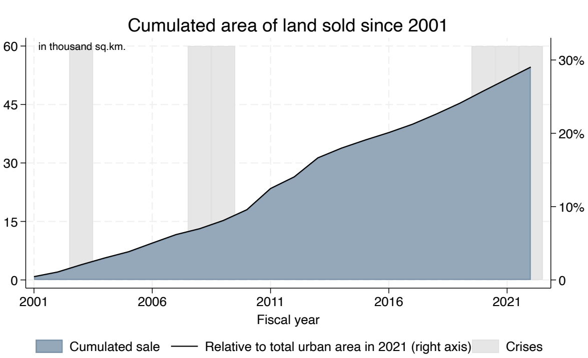
(a) Singapore, 2010-2022



(b) Hong Kong, 1985-2032 (with 10-year forecast)



(c) Mainland China, 2001-2022



Notes: (a) Singapore data since 2010 are from the aggregate statistics published on Singapore Land Authority's website. Detailed transfer data by sites going back to last century can be found in websites of various government agencies (Urban Redevelopment Authority, Jurong Town Corporation, and Housing Development Board). (b) Hong Kong land sale records and forecasts are from the Hong Kong Lands Department. (c) Mainland China data are from National Bureau of Statistics and Ministry of Natural Resources.

Stylized fact 2. (Gradual land supply) *While much land remains unused, governments only incrementally release land for private sector use.* These Asian growth miracles all gradually and steadily expand land supply to the private sector, illustrated in Figure 2. Importantly, the expansion is not constrained by the availability of land; rather, much land sits idle, indicated by Figure A2. Similarly, hundreds of millions of acres of land were only gradually transferred by the US governments to private individuals and businesses over the course of 19th century (Anderson and Martin, 1987; Libecap, 2007).[3]

I provide a theory that speaks to both stylized facts. The latter fact concerning land supply, seemingly straightforward, warrants some further discussion. While smooth consumption of exhaustible resources such as oil does call for gradual extraction and use, since land does not depreciate, its smooth consumption prescribes the same amount of land used in each point of time. Hence there are two aspects worth noting separately—the government chooses to have some land idle and it gradually increases land supply.

This paper presents three main sets of results, summarized in Table 2. First, in Section 2, I develop a model of land finance and investment-led growth, designed to be close to the neoclassical growth model. Production uses public capital as an external input, and the economy grows through capital accumulation. Public capital is assumed to be public good provided by the government à la Barro (1990), hence a role of a benevolent government. A representative household derives utility from non-durable good and land, with the only use of land being consumption. The economy is endowed with one unit of land and part of it is owned by the government. The government can sell or lease land to the household to raise public funds for investment. As land does not depreciate, its value embeds future economic prosperity which is enhanced by the provision of public capital, capturing the intuition from Tideman et al. (1990). Section 3.2 sets up the Ramsey planning problem for a benevolent government and characterizes the optimal allocation (second best, SB henceforth). Assuming complete markets and that the government can commit to its future actions, I show that the SB allocation features a public-investment-to-GDP ratio that declines over time (e.g., front-loaded investment) and a constant total land supply. When the demand for land is *inelastic*, the optimal policy leaves some land idle to generate high profits for public investment at the cost of reduced land consumption. Notably, my analysis leads to a novel quantity-based variant of the well-known Lerner formula, which holds under *inelastic demand* and prescribes a lower land supply when

[3]To this day, the US federal and state governments still own nearly 40% of land nationwide, including over 15% in eastern states such as New Jersey, New York, and Pennsylvania (Figure A1).

the demand elasticity is lower or the social value of public fund is higher. In this SB benchmark, the land supply is time-invariant. The intuition is that, given the amount of money the government needs to raise in present value, it does so in the least distortionary way, hence a stationary land supply.

Table 2: Summary of results

	Complete markets	Borrowing constraint
Commitment	Second-best (SB) allocation: front-loaded investment and *constant* land supply	SB can be achieved
Discretion	SB can be achieved	SB cannot be achieved; front-loaded investment and *rising* land supply $\rightarrow$ land contract design

Admittedly, complete markets and full commitment are both extremely strong assumptions, and the SB allocation featuring constant land supply is at apparent odds with the observed continual expansion. The second set of results pertains to analyzing the implications of frictions. I demonstrate in Section 3.3 and Appendix D that the SB allocation can be implemented with a balanced budget (no borrowing by either the government or the household), or in a time-consistent way. In particular, to implement SB with a balanced budget, the government cannot hold on to all the land forever, in contrast to Tideman et al. (1990)'s policy recommendation. This is because leasing land only gives a stationary fiscal-income-to-GDP ratio, which necessitates government borrowing since the SB investment-to-GDP ratio is front-loaded. Instead, the government should gradually sell the land it holds to front-load some future rent income to fund the SB investment. In Section 4, I establish that in the presence of *both* borrowing constraints and discretion, the government will indeed keep increasing land supply. The intuition is that a discretionary government always takes the amount that is already sold as given and wants to sell more to fund investment, a result conforming to the Coase (1972) conjecture. In Appendix E, I present suggestive evidence in a panel of Chinese cities that the interaction of discretion and financial constraints leads to more land supply.

Last, as the continual expansion of land supply indicates a deviation from the SB allocation and thus lower social welfare, is there a remedy? In Section 5, I propose an innovative land contract that links land supply to the provision of public capital to discipline future governments. Specifically, the optimal land contract penalizes future governments' deviation by obliging them to give more land to the holder of such contracts if the public capital stock is lower, per unit of contract outstanding. This contract, whose institutional requirement is the same as administrating land sale, presents itself as a practical policy option to improve social welfare.

1.1 Literature review

This paper connects to the literature on optimal fiscal policy since Ramsey (1927), which is primarily concerned with taxation and typically assumes an exogenous stream of government expenditure. I build on Barro (1990)'s growth model, incorporating the role of productive public investment to endogenize government expenditure.[4] Kydland and Prescott (1977) first note potential time inconsistency of optimal policy. Lucas and Stokey (1983) show that in a model without capital, under complete markets, the optimal fiscal policy to finance exogenous spending (à la Ramsey) can be time-consistent. Debortoli, Nunes and Yared (2021) further qualify this statement. In my model, the time inconsistency problem stems from the government being a durable good monopoly à la Coase (1972). Drawing on techniques of Lucas and Stokey (1983) and Debortoli, Nunes and Yared (2021), I show that the optimal policy in my model which involves time-invariant land supply can be implemented in a time-consistent manner, thus shedding new light on the Coase (1972) conjecture. I draw connections of the optimal land supply being time-invariant in my model to the tax smoothing result à la Barro (1979) and Aiyagari et al. (2002) of optimal taxation. Angeletos (2002) and Buera and Nicolini (2004) investigate optimal fiscal policy under commitment with non-contingent debt only. Debortoli, Nunes and Yared (2017, 2022) instead study optimal fiscal policy under complete markets in the absence of commitment. These papers make use of quantitative methods to arrive at some of their results. I offer analytical insights into the optimal policy in my model when removing either commitment or financial markets or both. While I highlight various conceptual links to this literature, this paper differs prominently in its subject of study—the role of land in government revenue instead of tax and the design of land contract—and endogenously determines the stream of public spending rather than taking it as given.

A related strand of literature analyzes optimal development policies. Acemoglu,

[4] Ratner (1983), Aschauer (1989) and Munnell (1990*a*,*b*) seminally estimate the output elasticity of public capital. See Bom and Ligthart (2014) for a review and Leduc and Wilson (2012) and Ramey, Glaeser and Poterba (2021) for recent empirical advances.

Aghion and Zilibotti (2006) study the optimal policy choice regarding technology adoption and innovation, which depends on the distance from the world technology frontier. Itskhoki and Moll (2019) consider optimal development policies when entrepreneurs face borrowing constraints. Liu (2019) characterizes optimal industrial policies in production networks with distortions. These papers and mine each examine a different aspect of economic development. My analysis emphasizes the funding of government expenditure, and features land (the revenue source) as an asset whose current value reflects future economic growth.

It is also of interest to compare land to exhaustible resources such as oil and minerals. The study of exhaustible resources was once a prominent topic of economic research (Hotelling, 1931; Solow, 1974). Endowment of such resources also has profound implications for economic growth or lack thereof, known as the "resource curse" (Sachs and Warner, 1995). While both land and oil are valuable resources, this paper suggests that the nature of land and thus its optimal management differs significantly. As land does not depreciate whereas used oil is gone, consumption smoothing of land calls for a time-invariant supply—a constant stock rather than flow. Moreover, the demand of land is local and endogenous to the country's growth, while the demand for oil is oftentimes from an international market. In practice, only a few countries are fortunate (or burdened) with significant endowments of exhaustible resources, whereas land management poses a challenge for every nation at various points in time.

There has been a lot of attention and work on the practice of land finance, especially in the context of China. It is commonly agreed that land finance has played a key role in China's growth miracle, and bears important implications for the Chinese real estate market (the largest asset class in the world).[5] Many empirical papers establish interesting facts; see Zheng et al. (2014), He et al. (2023), and Chang, Wang and Xiong (2023), among others. A few papers embed fixed rules of land supply and government budget into quantitative models (for example, Liu, 2018*a*; Jiang, Miao and Zhang, 2022; Wen and Jin, 2022). Differing from them, this paper offers an analytical theory to understand land finance as a policy choice. In particular, it speaks to the stylized facts shared by several economies regarding infrastructure investment and gradual expansion of land supply, rather than taking them for granted. My analysis of government discretion and

[5] For a comprehensive review of China's land system and economic development, see Liu (2018*b*). Gyourko et al. (2022) provide a recent survey of the literature on China's land finance. See for example Fang et al. (2016), Glaeser et al. (2017) and Liu and Xiong (2020) on China's real estate market.

borrowing constraints derives lessons that are broadly applicable and not restricted to China.

2 The economy and first-best allocation

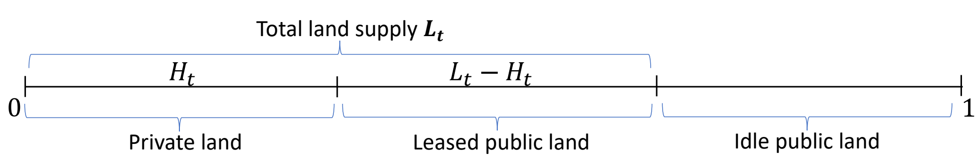
2.1 The economy

The economy is fully deterministic. An infinitely-lived representative household derives utility from (non-durable) consumption C_t and housing/land L_t

$$\mathcal{U}_0 \equiv \int_0^\infty \mathrm{e}^{-\rho t} U(C_t, L_t)\,\mathrm{d}t = \int_0^\infty \mathrm{e}^{-\rho t}\left(\ln C_t + \nu \frac{L_t^{1-\sigma}-1}{1-\sigma}\right)\mathrm{d}t, \tag{1}$$

with $\rho > 0, \sigma > 1$. The assumption of $\sigma > 1$ ensures that the demand for housing, holding fixed consumption C, is inelastic, as will become clear later. Housing is equivalent to land in this economy, and I will use these two terms interchangeably. I normalize the total amount of land in this economy to one without loss of generality. Time-t housing consumption consists of H_t amount of owned land that is valued at price P_t, and $L_t - H_t$ amount leased from the government at rent D_t, as illustrated in Figure 3.[6] At time t, the household could also buy or sell housing at rate $\dot{H}_t$ at the prevailing price P_t which will determine the owned amount $H_{t+\mathrm{d}t}$ in the next instance. Land does not depreciate.

Figure 3: Land utilization in the model



The household supplies one unit of labor, $N = 1$. The production uses public capital stock Z_t with total factor productivity (TFP) A.[7] The stock Z_t depreciates at rate $\delta \geq 0$. For concreteness, I may refer to the public capital as infrastructure, but one may also

[6] The US government both sells and leases land. Other governments (e.g., Singapore, China) transfer land via long-term and short-term leases. In the model, allowing the government to both sell and lease capture these various forms of land contracts. Section 3.3 offers a more detailed discussion.

[7] Growth in TFP and population will be accommodated in an extension considered in Appendix **??**, but they do not change the qualitative prediction that the second-best allocation features *constant* amount of land supplied to the private sector.

include other forms of capital or public service with a higher δ. For the production, the household takes Z_t as an external input (i.e. infrastructure as a public good), which is the only externality in this model. The household earns labor income Y_t. The infrastructure investment is thus carried out by the government. The time-t resource constraint is

$$C_t + \dot{Z}_t + \delta Z_t \leq Y_t(Z_t) = AZ_t^{\alpha} N, \tag{2}$$

with $\alpha \in (0,1)$. The production function is of a neoclassical nature, with an incorporation of public investment à la Barro (1990).[8] I denote the consumption rate (consumption-to-GDP ratio) as

$$\chi_t \equiv \frac{C_t}{Y_t}$$

and thus the investment rate (investment-to-GDP ratio) is $\frac{\dot{Z}_t + \delta Z_t}{Y_t} = 1 - \chi_t$. Our first stylized fact that government-investment-to-GDP ratio declines as economic growth slows down points to χ_t increasing over time (and thus $1-\chi_t$ decreasing).

The economy starts at time 0 with public capital stock Z_0 and land in private hands of the amount $H_0 < 1$. The government has no debt when the economy starts, and is endowed with the rest of the land $(1 - H_0)$. At each time t, the amount of land the government supplies L_t and the household owns H_t satisfy

$$L_t \in [0,1], \tag{3}$$

$$H_t \in [0,1]. \tag{4}$$

The government derives income from land lease and sale as their only revenue source to fund government investment. The government is allowed to rebate extra money to the household in a lump-sum manner (but they may not want to), but cannot levy lump-sum tax.

In this section, we assume complete markets, and we will introduce borrowing constraints in later sections.

[8] Xiong (2019) uses a similar specification of production function to analyze the role Chinese government played in its growth miracle. His focus is on the agency problem between the central and local governments, whereas this paper studies the dynamic problem of land supply and public investment.

2.2 Remarks on model assumptions

Before presenting results, we provide some remarks on the model. This model is designed to minimize deviation from a neoclassical growth model while offering insights into land finance. Appendix **??** offers a few formal extensions.

1. The only source of inefficiency is that the household takes public capital Z_t as given, which gives rise to a role of a benevolent government. We will show that, in order to generate more profits, the government may lower land supply. This generates a tradeoff between fundraising and welfare loss from reduced land consumption.

2. I assume the land is exclusively designated for housing purposes. In reality, land is used for production too and is typically zoned for different uses. The insight that a less-than-maximum supply of land ($L_t < 1$) may be chosen to increase public revenue even if it incurs welfare cost should be orthogonal to the exact use of land. Appendix **??** considers land used as factor of production instead of housing. I further assume that building infrastructure does not use land, which is harmless if the land used for infrastructure does not exceed the amount of land optimally left idle (i.e., not consumed), which holds true in reality (Figure A2).

3. I assume away unrealistic lump-sum tax. Otherwise the government will be able to impose Pigouvian tax to pay for public investment directly. I assume away land tax for simplicity too. Pure land tax is practically non-existent. Property tax exists, but it distorts investment in housing structure, and is hard to implement (especially for developing countries) as it requires pricing off-market houses. An insight behind the celebrated Henry George theorem is that land tax is *non-distortionary* as land is in fixed supply. In Appendix C.1, I suggest a limit of this logic: while land tax is non-distortionary, it may be *insufficient* to fund "best outcome" if the tax base (total land market value) is low. The incentive analyzed in this paper that the government may lower land supply in order to raise its market value can still play a role, in which case there will be welfare loss from reduced land utilization, even if the government can collect land tax.

4. I abstract away from various distortionary taxes for simplicity. This model could be augmented to feature endogenous labor supply and income tax. In that case, it is desirable to use both instruments (labor tax and land supply) to raise funds and balance between multiple sources of welfare loss (labor supply distortion and

lowered land utilization). A further reason not to include private capital and capital tax is that the time consistency issue with capital tax is well-known. Shutting that down helps isolate the time consistency issue with land. A large literature since Ramsey (1927) reviewed in Section 1.1 studies distortionary taxation alone, while this paper focus on land. Future work may explore their interactions.

5. This model is interesting only when there is a non-trivial amount of land owned by the government at time 0. This is the case for the US and Hong Kong due to their colonial histories. For other economies this model can be viewed as analyzing the policy problem *after* the government acquires land from the private sector. In Singapore, the government already owned 44% of land in 1960 (before independence), and it passed the Land Acquisition Act in 1966 (after independence) to acquire more land for public purpose at a fixed price. In mainland China, the government acquires land from rural collectives (groups of local farmers) and "rezones" it as urban land. Through acquisitions, a government can obtain the land, engage in infrastructure investment which raises the land value, and thus profit from the appreciation of land. We directly place ourselves in the second stage where the government already has land at its disposal.

2.3 First-best (FB) allocation

The first-best (FB) allocation maximizes household utility (1) subject to the resource constraint (2) and land constraints (3, 4).

Proposition 1. (First best allocation) *The FB allocation* $\left(C_t^{FB}, Z_t^{FB}, L_t^{FB}, H_t^{FB}\right)$ *is completely characterized as follows, given* Z_0*:*

1. *the consumption growth follows*

$$\frac{\dot{C}_t^{FB}}{C_t^{FB}} = Y_t'\left(Z_t^{FB}\right) - (\delta + \rho), \tag{5}$$

subject to the transversality condition and the resource constraint

$$\dot{Z}_t^{FB} = Y_t\left(Z_t^{FB}\right) - C_t^{FB} - \delta Z_t^{FB}; \tag{6}$$

2. *the land supply is maximum*

$$L_t^{FB} = 1; \tag{7}$$

3. *the amount of land held in private hands* H_t^{FB} *for* $t > 0$ *can take any value in* $[0, 1]$.

The path of consumption and public capital at the first-best allocation in this economy is identical to the path of consumption and (private) capital in a neoclassical growth model. However, the market outcome in a neoclassical model coincides with the first best, whereas this economy embeds a market failure that the household takes the public capital as given, which may inhibit the FB allocation.

Under constant TFP A, the economy converges to a steady-state that features

$$Y'\left(Z_\infty^{FB}\right) = \delta + \rho \quad \Rightarrow \quad Z_\infty^{FB} = \left(\frac{A\alpha}{\delta + \rho}\right)^{\frac{1}{1-\alpha}}. \tag{8}$$

At the steady state, the consumption rate is

$$\chi_\infty^{FB} = 1 - \frac{\delta Z_\infty^{FB}}{Y_\infty^{FB}} = \frac{\delta\,(1-\alpha) + \rho}{\delta + \rho} \tag{9}$$

and the investment rate is thus its complement $\frac{\delta\alpha}{\delta+\rho}$.

3 Second-best allocation and constrained implementations

Now we restrict attention to allocations that can be sustained as equilibrium outcomes. Section 3.1 studies household optimization. Section 3.2 sets up the Ramsey planning problem in primal form for the government, which will give rise to the second-best (SB) allocation, under the assumptions of complete financial markets and commitment by the government. Section 3.3 relaxes these assumptions, and shows that the SB allocation can still be implemented.

3.1 Household behavior and premises for second best (SB)

We assume that the household borrowing is unconstrained other than by the transversality condition. Thus the household is subject to a present-value budget constraint, where Q_t

denotes the bond price under normalization $Q_0 = 1$,

$$
\begin{aligned}
0 &= \int_0^\infty Q_t \left[C_t - Y_t(Z_t) + P_t \dot{H}_t + D_t(L_t - H_t) - T_t\right] \mathrm{d}t \\
&= \int_0^\infty \left\{ Q_t \left[C_t - Y_t(Z_t) + D_t(L_t - H_t) - T_t\right] - H_t \frac{\mathrm{d}(P_t Q_t)}{\mathrm{d}t} \right\} \mathrm{d}t - P_0 H_0
\end{aligned} \tag{10}
$$

where we normalize $Q_0 = 1$.

The household optimality conditions pin down bond price and land rent as

$$Q_t = \mathrm{e}^{-\rho t} U_{C,0}^{-1} U_{C,t} \tag{11}$$

$$D_t = U_{C,t}^{-1} U_{L,t} \tag{12}$$

and thus the land price satisfies

$$P_t = Q_t^{-1} \int_t^\infty Q_s D_s \mathrm{d}s = U_{C,t}^{-1} \mathcal{U}_{L,t} \tag{13}$$

in which $\mathcal{U}_{L,t} \equiv \int_t^\infty \mathrm{e}^{-\rho(s-t)} U_{L,s} \mathrm{d}s$ encodes the discounted sum of marginal utility from housing. Land rent D_t declines in current supply L_t, due to diminishing marginal housing consumption utility. Land price P_t decreases in future supply $L_{\tau \geq t}$ too, since land does not depreciate. Thus the government is a durable good monopolist à la Coase (1972) and faces time inconsistency issue: in order to raise current price P_t, the government wants to hold future supply low; but when future comes, there is an incentive to deviate since what is sold is sold.

Holding fixed the path of land supply $\{L_t\}$, land price increases in consumption C_t: as the economy grows but land is ultimately in fixed supply, land price appreciates. Having fiscal income derived from land allows the government to invest in public capital Z_t and helps the economy growth. Hence, there is a positive feedback between land finance and economic growth the government can activate, capturing the intuition from Tideman et al. (1990). This paper discusses extensively its use and misuse.

Now we impose assumptions on our problem such that the first best cannot be achieved as market outcome, and that the government may profit from restricting land supply.

Fiscal requirement of FB. We assess the fiscal position evaluated at the market price determined by household optimization (11-13) under the FB allocation. The present-value of time-t government investment in infrastructure, multiplied by $U_{C,0}$ to transform into unit of utility, is

$$\mathcal{E}_t^{FB} = Q_t \left(\dot{Z}_t^{FB} + \delta Z_t^{FB} \right) U_{C,0} = \mathrm{e}^{-\rho t} U_{C,t} \left(\dot{Z}_t^{FB} + \delta Z_t^{FB} \right) = \mathrm{e}^{-\rho t} \left(\frac{1}{\chi_t^{FB}} - 1 \right),$$

where the last step uses the resource constraint (2) and the consumption rate $\chi_t^{FB} = \frac{C_t^{FB}}{Y_t^{FB}}$. The present-value util-unit of total government spending is thus

$$\mathcal{E}^{FB} = \int_0^\infty \mathcal{E}_t^{FB} \mathrm{d}t = \mathcal{E}^{FB} \left(Z_0 \right),$$

which implicitly depends on the initial infrastructure stock Z_0.

Holding $H_t = H_0$ fixed,[9] the government derives income from land lease at each time t whose present value, again multiplied by $U_{C,0}$ to transform into util, is

$$\mathcal{I}_t^{FB} = Q_t D_t \left(1 - H_0 \right) U_{C,0} = \mathrm{e}^{-\rho t} \nu \left(1 - H_0 \right),$$

and the present-value util-unit of total government income is

$$\mathcal{I}^{FB} = \int_0^\infty \mathcal{I}_t^{FB} \mathrm{d}t = \frac{\nu \left(1 - H_0 \right)}{\rho},$$

Assumption 1. The following inequalities imply that the FB cannot be achieved

$$Z_0 < Z_\infty^{FB} = \left(\frac{A\alpha}{\delta + \rho} \right)^{\frac{1}{1-\alpha}}, \tag{14}$$

$$\nu \left(1 - H_0 \right) < \rho \int_0^\infty \mathrm{e}^{-\rho t} \left(\frac{1}{\chi_t^{FB}} - 1 \right) \mathrm{d}t. \tag{15}$$

Inequality (14) in Assumption 1 states that the economy starts with a low infrastructure stock. Otherwise the FB allocation can be trivially achieved by consuming existing infrastructure stock. The second inequality ensures that if the government supplies all the land to the private sector ($L_t = 1$), it gets insufficient amount of fund compared to the required FB investment. The left-hand side of the inequality involves the initial amount of land held by the household H_0 and housing utility parameter ν, and the right-hand

[9] In fact, the claim that follows holds even if H_t varies over time, because the housing price is "fair" in incorporating all rental revenue when household borrowing is unconstrained.

side implicitly depends on the non-durable consumption part of the model (parameters $Z_0, A, \beta, \alpha, \rho$). This clean separation is made possible by the separably utility, and is a property that helps keep much of our later analysis tractable. Under Assumption 1, the government is too poor relative to FB, and thus the optimal lump-sum transfer T_t will be zero.

Monopoly land supply. Assumption 1 ensures that the FB allocation cannot be achieved with maximum land supply $L_t = 1$. We further focus on a case which is also empirically relevant where the government can raise more money by restricting land supply.

We first take a small detour to derive a useful variant of the celebrated Lerner rule, which is inspired by but extends beyond this model. This result will also provide useful intuition for our later results.

Lemma 1. (A quantity Lerner rule) *Consider a monopoly that faces a demand curve $q(p)$. Assume that it produces with zero marginal cost, but the buyer has already owned some quantity q_0, so the monopoly profit is*

$$\max \pi = (q(p) - q_0)\, p \tag{16}$$

The supply to maximize profit π satisfies

$$\frac{q - q_0}{q} = \underbrace{-\frac{\mathrm{d}\log q(p)}{\mathrm{d}\log p}}_{\text{demand elasticity}} \tag{17}$$

Formula (17) suggests that, in problem (16), a solution with finite quantity exists when the demand is *inelastic*. This is meaningful as decades of empirical research almost all point to inelastic demand for housing.[10] This requirement is in contrast to the textbook Lerner rule (Tirole, 1988, Chapter 1.1) which gives rise to a solution only when demand is elastic. The high-level difference is that it is a pre-owned quantity q_0 that erodes the monopoly's profit in problem (16), rather than a positive marginal cost as in the textbook case. Appendix C.2 discusses this in greater detail. While our quantity Lerner rule works under inelastic demand and the textbook Lerner rule is only applicable to elastic demand, they share qualitatively seminar comparative statics: the supply q increases in the demand elasticity and the price p decreases in the demand elasticity.

[10] Albouy, Ehrlich and Liu (2016) estimate an elasticity of about two thirds and review earlier estimates.

Coming back to our analysis of land finance, taking household demand of housing (12), we can derive the demand elasticity of housing $-\frac{\partial \log L_t}{\partial \log D_t} = \sigma^{-1}$, which is less than one under our assumption $\sigma > 1$. That is, an ϵ percentage demand shock to the housing market changes the price of housing by $\sigma\epsilon > \epsilon$ percentage, which is the empirically realistic case. If the government were to maximize its time-t profit from land lease, i.e., $\max_{L_t} (L_t - H_t) D_t (L_t)$, the optimal land supply would satisfy from Lemma 1, if interior,

$$\frac{L_{mono} - H_t}{L_{mono}} = -\frac{\partial \log L_t}{\partial \log D_t} = \sigma^{-1} \tag{18}$$

Formula (18) admits a solution $L_{mono} = \frac{\sigma}{\sigma-1} H_t$ since $\sigma > 1$. When $\frac{\sigma}{\sigma-1} H_t$ is less than one, a government that maximizes profit would set $L_{mono} = \frac{\sigma}{\sigma-1} H_t < 1$. Later, when we introduce a benevolent government that maximizes the household's utility, it would balance the need to raise money for investment against the household utility gain from living in larger houses.

Assumption 2. The following inequality implies that the government can profit from restricting land supply

$$\frac{\sigma}{\sigma - 1} H_0 < 1. \tag{19}$$

We make Assumptions 1 and 2 henceforth. They can hold simultaneously, since 1 hinges on ν while 2 relates to σ, in addition to the dependence of both on H_0.

3.2 Optimal policy under commitment and complete markets

Using the household optimality conditions (11-13) to substitute prices in the budget constraint (10), as is a standard technique in the literature since Lucas and Stokey (1983), we get the following implementability condition

$$0 = \int_0^\infty \mathrm{e}^{-\rho t} \left[(C_t - Y_t (Z_t) - T_t) U_{C,t} + (L_t - H_0) U_{L,t} \right] \mathrm{d}t. \tag{20}$$

The absence of P_t and H_t in this condition results from the "fair price" of housing as the household faces no borrowing constraints. The discounted sum of payment to housing consumption L_t is the same, regardless of how much the household buys or rents. When both the implementability constraint (20) and the resource constraint (2) holds, the government's budget constraint holds as a result of the Walras' law.

Definition 1. *(Ramsey problem under commitment and complete markets)* A Ramsey planner maximizes the household's life-time utility (1), subject to the resource constraint (2), land constraint (3), as well as the implementability constraint (20).

I attach multiplier $\mu_t^* e^{-\rho t}$ to the time-t resource constraint (2), $\zeta_t^* e^{-\rho t}$ to the time-t land constraint (3), and multiplier λ^* to the implementability constraint (20). λ^* measures the *social value of public fund*. I use superscript * to indicate the optimal allocation and policy, *decided at time* 0 *under commitment*. Formally, the Lagrangian is

$$\mathcal{L}^* = \max_{\{C_{t\geq 0}, L_{t\geq 0}, T_{t\geq 0}, H_{t>0}, Z_{t>0}\}} \int_0^\infty e^{-\rho t} U(C_t, L_t)\, dt + \int_0^\infty e^{-\rho t} \{\mu_t^* [Y_t(Z_t) - C_t - (\delta+\rho) Z_t] + \dot{\mu}_t^* Z_t\}\, dt + \mu_0^* Z_0$$
$$+ \int_0^\infty e^{-\rho t} \zeta_t^* (1 - L_t)\, dt + \lambda^* \int_0^\infty e^{-\rho t} [(C_t - Y_t(Z_t) - T_t) U_{C,t} + (L_t - H_0) U_{L,t}]\, dt \tag{21}$$

where we have used the present-value sum version of capital stock in place of its flow version for a more intuitive representation.[11]

The SB policy satisfies

$$0 = e^{\rho t} \frac{\partial \mathcal{L}^*}{\partial C_t} = (1+\lambda^*) U_{C,t} + \lambda^* (C_t - Y_t) U_{CC,t} - \mu_t^* = U_{C,t} - \lambda^* Y_t U_{CC,t} - \mu_t^* \tag{22}$$

$$0 = e^{\rho t} \frac{\partial \mathcal{L}^*}{\partial L_t} = (1+\lambda^*) U_{L,t} + \lambda^* (L_t - H_0) U_{LL,t} - \zeta_t^* \tag{23}$$

$$0 = e^{\rho t} \frac{\partial \mathcal{L}^*}{\partial Z_t} = \mu_t^* [Y_t' - (\delta+\rho)] + \dot{\mu}_t^* - \lambda^* Y_t' U_{C,t} \tag{24}$$

We first observe that H_t appears in neither the objective (1) nor the constraints (2, 3, 20). Thus, given the total supply $\{L_t^*\}$, the division between sale and lease is indeterminate under SB, formalized in Lemma 2. The reason behind is that as households optimize freely intertemporally, the housing price P_t depends on the total supply L_t but not on the owned quantity H_t and the household is willing to hold any quantity $H_t \in [0,1]$ at the market price. However, the level of H_t will matter for the amount of borrowing at each point in time between the household and the government, as sale can substitute for borrowing in raising fund for the government.

Lemma 2. (Indeterminate sale/lease ratio under SB) *The optimal policy may feature any* $H_t \in [0,1]$.

[11] See derivation of (**??**) in Appendix **??**.

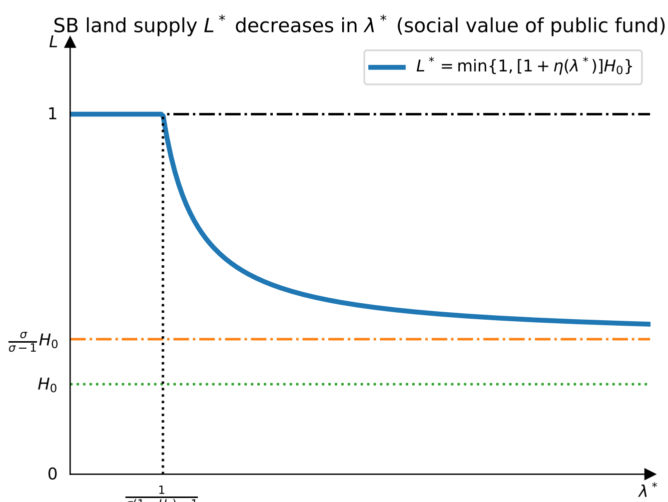
Using the FOC w.r.t. total land supply L_t (23), we can characterize the optimal land supply which equals $L^* = \min\left\{1, L^*_{interior}\right\}$ with

$$\frac{L^*_{interior} - H_0}{L^*_{interior}} = \left[1 + (\lambda^*)^{-1}\right]\sigma^{-1}. \tag{25}$$

$L^*_{interior}$ decreases in both the social value of public fund λ^* and σ, depicted in Figure 4. Comparing the SB land supply (18) to the land supply that would maximize the profits from land (25), the former converges to the latter when $\lambda^* \to \infty$. In that case, an additional dollar to the government leads to an infinite increase of social welfare and thus the benevolent government acts as if it maximizes profits. In the other extreme, if $\lambda^* \to 0$, it is guaranteed that $L^* = 1$: if a transfer to the government does not raise social welfare, the government should not make money by restricting land supply that hurts housing consumption.

Indeed, there exists a threshold $\bar{\lambda} \equiv \frac{1}{\sigma(1-H_0)-1}$ which is positive under Assumption 2 such that the optimal land supply is interior if and only if $\lambda^* > \bar{\lambda}$. When the marginal welfare of an extra dollar to the government is small ($\lambda^* < \bar{\lambda}$), the government prefers not to distort the land consumption. We could see this by evaluating (23) at $L_t = 1$ to get $e^{\rho t} \frac{\partial \mathcal{L}^*_0}{\partial L_t}\Big|_{L_t=1} = (1+\lambda^*)\, U_{L,t} + \lambda^* (1-H_0)\, U_{LL,t}$ which is positive if λ^* is small. When the marginal welfare improvement λ^* of government making money is small, the government avoids welfare loss from reduction in housing consumption $U_{L,t}$.

Figure 4: Optimal land supply in the model



Lemma 3. (Land supply under SB) *The optimal land supply is time-invariant and satisfies*

$$L^* = \begin{cases} 1, & \lambda^* \leq \bar{\lambda} = \frac{1}{\sigma(1-H_0)-1}, \\ L^*_{interior} = [1 + \eta(\lambda^*)] H_0 < 1, & \lambda^* > \bar{\lambda}, \end{cases} \tag{26}$$

with $\eta(\lambda^*) = \left(\frac{\lambda^* \sigma}{1+\lambda^*} - 1\right)^{-1}$ *which is decreasing in* λ^* *denoting the supply in excess of initial household-owned land* H_0.

At the interior solution, the optimal land supply is time-invariant. That is because the utility $U(C, L)$ is separable and hence the convergence of consumption C_t does not alter the marginal utility from housing $U_{L,t}$. It is thus optimal to smooth out the housing consumption, much like the tax-smoothing result from Barro (1979).

To focus on an interior solution, henceforth we make Assumption 3 as follows, which implies that the SB land supply is interior based on Lemma 3. Assumption 3 supersedes Assumptions 1 and 2.

Assumption 3. *(Interior SB)* Assume (Z_0, H_0) are such that $\lambda^* > \frac{1}{\sigma(1-H_0)-1}$.

So far we have analyzed the land supply. Making use of the FOCs w.r.t. consumption C_t and public capital Z_t (22, 24), we can derive the path of consumption. Further, λ^* is determined by the implementability condition (20). Proposition 2 completely characterizes the SB allocation, whose proof is in Appendix B.

Proposition 2. (Interior SB allocation) *The SB allocation* $\left(C^*_t, Z^*_t, L^*_t, H^*_t\right)$ *is completely characterized as follows, given* (Z_0, H_0)*:*

1. *the consumption growth follows*

$$\frac{\dot{C}^*_t}{C^*_t} = \left[Y'_t(Z^*_t) - (\delta + \rho)\right] - \frac{\lambda^*}{\lambda^* + \chi^*_t}\left[\chi^*_t Y'_t(Z^*_t) + \frac{\dot{\chi}^*_t}{\chi^*_t}\right], \tag{27}$$

with consumption rate $\chi^*_t \equiv \frac{C^*_t}{Y^*_t}$ *that strictly increases over time, subject to the transversality condition and the resource constraint*

$$\dot{Z}^*_t = Y_t(Z^*_t) - C^*_t - \delta Z^*_t; \tag{28}$$

2. *the land supply is time-invariant and satisfies*

$$L^* = [1 + \eta(\lambda^*)] H_0, \tag{29}$$

with excess supply function $\eta(\lambda) \equiv \left(\frac{\lambda\sigma}{1+\lambda} - 1\right)^{-1}$ *which decreases in* λ*;*

3. *the amount of land held in private hands* H_t^* *for* $t > 0$ *can take any value in* $[0, 1]$*;*

4. *the multiplier* λ^* *is positive and satisfies*

$$\nu H_0^{1-\sigma} \frac{\eta(\lambda^*)}{[1 + \eta(\lambda^*)]^{\sigma}} = \rho \int_0^{\infty} \mathrm{e}^{-\rho t} \left(\frac{1}{\chi_t^*} - 1\right) \mathrm{d}t \tag{30}$$

whose LHS is increasing in λ^**.*

The consumption growth process (27) coupled with the resource constraint (2) is a family of ordinary differential equations (ODEs) parameterized by $\lambda^* > 0$ that meets the boundary condition Z_0 and the transversality condition. The limit of $\lambda^* \to 0$ describes the FB allocation (Proposition 1). When λ^* is positive, there is a wedge between the net marginal product of public capital ($Y_t'\left(Z_t^*\right) - \delta - \rho$) and consumption growth rate $\dot{C}_t^*/C_t^*$. That is a result of the government optimally lowering the private rate of return to fund its investment.

Further, extending a result from Barro and Sala-i-Martin (2004, Chapter 2.6), I show that χ_t^* strictly increases during convergence. Barro and Sala-i-Martin give conditions for monotonicity of χ_t^{FB} during transition under efficient allocation, i.e. the ODE system (27, 28) with $\lambda^* = 0$. I generalize their result to $\lambda^* \geq 0$. My result directly speaks to the stylized fact 1 regarding early public investment in this model. Even when the government cannot achieve the first best, it still chooses to front-load investment (investment rate $1 - \chi_t^*$ strictly decreasing over time as the economy grows).[12]

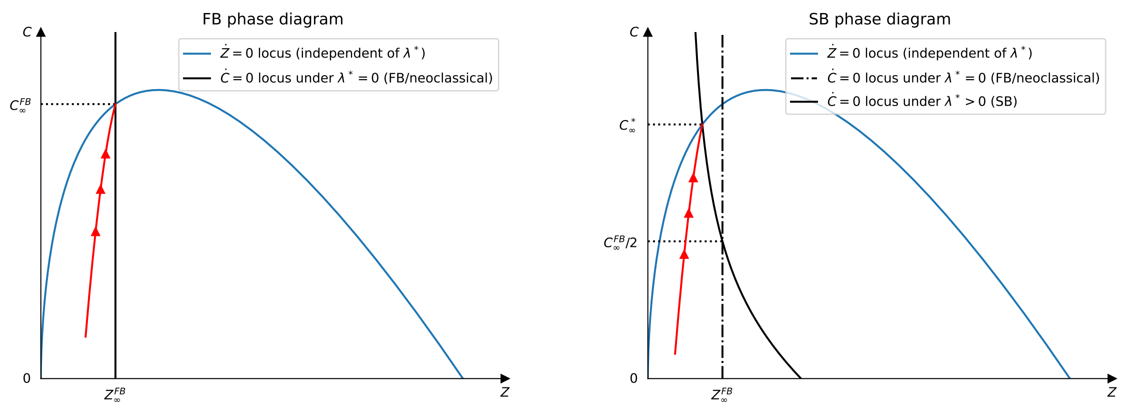
Based on (27, 28), Figure 5 plots the phase diagram in the (Z, C) space. The shape of $\dot{Z} = 0$ is independent of λ^*, as it arises from the resource constraint. When $\lambda^* = 0$, the $\dot{C} = 0$ locus is a vertical line, as in the neoclassical growth model. Denote the intersection of these two loci as $(Z_\infty^{FB}, C_\infty^{FB})$. When $\lambda^* > 0$, the $\dot{C} = 0$ locus rotates counterclockwise around

[12] I assumed log utility in my model for simplicity. This functional form can be relaxed without disturbing the monotonicity of χ_t^*, as long as the elasticity of intertemporal substitution (EIS) is lower than $1/\alpha$. This inequality likes holds with a large margin: Havranek et al. (2015) reviews a vast literature that estimates EIS which mostly produce estimates less than or around one; Bom and Ligthart (2014) survey estimates of output elasticity of public capital α and suggest an average estimate of 0.106.

exactly $(Z_\infty^{FB}, C_\infty^{FB}/2)$ and its interaction with the $\dot{Z} = 0$ locus shifts to the left. Indeed, we establish the following comparative statics for the steady state value.

Figure 5: Phase diagram

(a) First best (b) Second best



Corollary 1. *At the steady state, if the multiplier λ^* is higher, both the consumption C_∞^* and the infrastructure stock Z_∞^* are lower, but the consumption rate $\chi_\infty^* \equiv C_\infty^*/Y_\infty^*$ is higher.*

Specifically, steady-state consumption rate χ_∞^* follows

$$\chi_\infty^*(\lambda^*) = \frac{(1-\lambda^*) + \sqrt{(1+\lambda^*)^2 + \frac{4\delta\alpha}{\delta(1-\alpha)+\rho}\lambda^*}}{2} \frac{\delta(1-\alpha)+\rho}{\delta+\rho} \geq \chi_\infty^{FB} \tag{31}$$

which increases in λ^* unless $\delta = 0$ (in which case $\chi_\infty^* = 1$ obviously holds as infrastructure does not depreciate). The steady-state infrastructure and consumption satisfy

$$Z_\infty^*(\lambda^*) = \begin{cases} \left[\frac{\alpha}{(1+\lambda^*)\rho}\right]^{\frac{1}{1-\alpha}} & \delta = 0 \\ \left[\frac{A(1-\chi_\infty^*)}{\delta}\right]^{\frac{1}{1-\alpha}} & \delta > 0 \end{cases} \tag{32}$$

$$C_\infty^*(\lambda^*) = \begin{cases} A\left[\frac{\alpha}{(1+\lambda^*)\rho}\right]^{\frac{\alpha}{1-\alpha}} & \delta = 0 \\ A\chi_\infty^*\left[\frac{A(1-\chi_\infty^*)}{\delta}\right]^{\frac{\alpha}{1-\alpha}} & \delta > 0 \end{cases} \tag{33}$$

both of which decrease in λ^* regardless of δ.

When the marginal value of a dollar to the government is higher (higher λ^*), the economy converges to a steady state with lower consumption and lower infrastructure stock but higher consumption-to-output rate. Obviously, the steady state consumption and infrastructure are both lower under SB than under FB, and the consumption rate is higher, since the FB allocation can be viewed as the limit of $\lambda^* \to 0$.

The steady-state housing price $P_\infty = U_{C,\infty}^{-1} \mathcal{U}_{L,\infty}$ is subject to two offsetting forces. On the supply side, when λ^* is higher, land supply L^* is lower, which pushes up the housing price. However, as a higher λ^* also leads to lower consumption C_∞^*, the demand force depresses the housing price. One cannot simply take P_∞ as an indicator of welfare.

3.3 Constrained implementations of SB allocation

In Section 3.2, we allow the government to flexibly choose between land sale or lease, to borrow, and to commit to its future actions. Here we analyze what role each of these assumptions play. The assumption of commitment directly circumvents the time inconsistency issue from Coase (1972), by doing the planning problem only once at time 0. As for the roles of forms of land contracts and financial markets, recall that the investment-to-GDP ratio $1 - \chi_t^*$ strictly rises over time. Using (12), the land-rent-to-GDP ratio $\frac{D_t^*}{Y_t^*} = U_L^* \frac{C_t^*}{Y_t^*} = U_L^* \chi_t^*$ strictly declines. Thus the fiscal expenditure is front-loaded but the fiscal income from lease is back-loaded. As total fiscal income and expenditure are equal in terms of present value, the timing mismatch suggests that the government's lease income is less than expenditure early on. Hence the government has to transfer funds intertemporally. This can be done by borrowing (in which case having one bond is enough), or by selling rather than leasing land (which amounts to collecting all future rents from a piece of land today).

One may wonder if the optimal policy is changed once one of these assumptions is removed. I establish that land contract forms, financial markets, and the ability to commit, individually, are not essential for implementing the SB allocation. Here I only discuss the intuition and connections to the literature, leaving formal statements to Appendix D.

Implementation with a single land contract. The SB allocation can be implemented with a single land contract, i.e. either sale or lease. This is an immediate implication of Lemma 2 and Proposition 2, since H_t is indeterminate. In fact, it can be implemented by any form of land contract, as it will be priced in the financial markets. If the government sells L^* amount of land directly to the household at time 0, this maximizes the debt owed by the household and the government can save the proceedings. If the government keeps leasing L^* amount of land, it has to borrows against its future lease income.

An important piece of policy advice from Tideman et al. (1990) is that, if there is potential borrowing constraints faced by the household, the government could rent the land out on an annual basis. While it is intuitive that outright sale can be problematic under borrowing constraints, it takes a model to see if it is a good idea for the government to hold on to the land forever. On this proposal, the previous paragraph suggests that the government would run a fiscal deficit early on since the lease income is insufficient to fund expenditure. A slightest friction or reason against government deficit or borrowing could render this undesirable. Next, I show that there is a way to implement the SB allocation without necessitating government or private sector borrowing.

No-borrowing implementation. Proposition D1 establishes that the government can implement SB by selling land at a particular pace to exactly fund expenditure at any point of time. The premise is that, for each piece of land, selling it amounts to collecting all future rents at once. As discussed before, if the government sells L^* amount outright, it will collect too much money which they need to save. Intuitively, they can sell a small amount at each point in time to make up the difference between fiscal expenditure and lease income. The household will afford it without borrowing too, as the SB allocation satisfies the resource constraint and the Walras' law. This result applies if the government transfers land via a long-term lease instead of selling it.

Our finding can be compared to the need for a "war chest" from the tax smoothing result from Barro (1979) and Aiyagari et al. (2002), when fiscal expenditure varies. For example, if a war breaks out at time 0 but peace is anticipated later, a government will optimally borrow to fund the current expenditure without raising the present-day tax. In this paper, the fiscal expenditure varies too (in an endogenous way), but no borrowing is necessary to fund the front-loaded expenditure. This is because of the asset property of land: while the SB allocation calls for a time-invariant land supply, the government can fine-tune the time profile of land income by altering the amount of sale vs lease. Recognizing the dual nature of land as both a consumption good and an asset, smoothing the amount for consumption does not constrain the temporal distribution of land income.

Time-consistent implementation. The essence of the time inconsistency problem is that the monopoly at time t always wants to sell more relative to what is already sold before time t, rather than what is in the buyer's hand at time 0. A solution suggested by Coase himself and formalized by Bulow (1982) is that the monopoly only leases but never sells. That argument is developed in a partial equilibrium setting with an exogenous interest rate to discount future profits. We could examine this proposal in our setting too. From our previous analysis we know that only using lease involves government borrowing, which unfortunately invites its own time inconsistency issue in general equilibrium. The prior work by Lucas and Stokey (1983) and Debortoli, Nunes and Yared (2021) suggests that it takes a full maturity structure of debt (complete markets) to make the optimal fiscal policy time-consistent. Borrowing from their techniques, Proposition D2 shows that the SB allocation can be implemented in a time-consistent way too, under complete markets. The time-0 planner can leave a particular debt portfolio and sale/lease combination to incentivize future planners to adopt the time-0 optimal policy. The required debt portfolio always spans the full maturity structure, even if its present value equals zero if the government sets a specific sale/lease ratio to maintain a balanced budget as suggested by Proposition D1. Proposition D2 further unveils an insight about the sale/lease combination that the amount sold H_t should never exceeds $\frac{\sigma-1}{\sigma}L^*$, because given H_t the time-t planner will always set her optimal L to be larger than or equal to $\frac{\sigma}{\sigma-1}H_t$.

4 Optimal policy under discretion and financial constraints

We have established that land contracts, complete markets, and commitment, individually, are not essential prerequisites for the implementation of SB allocation. While in reality, governments do use various land contracts (sale and lease, or long-term and short-term leases), complete markets and commitment are both unrealistically strong assumptions. Here we show that the combination of discretion and financial constraints denies SB allocation. I call this case *double deviation (DD)* for simplicity.

I assume that the government has to run a balanced budget, using income from land sale and lease to exactly pay for investment at each point in time.[13] Thus we have two state variables (Z_t, H_t). To tractably model discretion, I take the limit of $\Delta t \to 0$ of a discrete-time Markov Perfect Equilibrium (MPE) in which each governor is in charge of for one period with period length Δt. Each governor takes into account how her choice of

[13] If the government is allowed to save but not borrow, I conjecture that it is optimal not to save. The logic is that expenditure is optimally front-loaded, and if the government sells more than it needs for expenditure, this exacerbates the time inconsistency problem.

(Z_t, H_t) affects the future governors' choices.

Now the budget constraint holds period by period as

$$0 = (C_t - Y_t(Z_t) - T_t)\Delta t + (L_t - H_t) D_t \Delta t + \Delta H_{t+\Delta t} P_t$$

Plugging in the discretized household optimality conditions (12, 13), we rewrite this as a period-t implementability constraint

$$0 = (C_t - Y_t(Z_t) - T_t) U_{C,t}\Delta t + (L_t - H_t) U_{L,t}\Delta t + e^{-\rho\Delta t}\Delta H_{t+\Delta t}\mathcal{U}^{DD}_{L,t+\Delta t} \tag{34}$$

with $\mathcal{U}^{DD}_{L,t} \equiv \sum_{s\geq 0} e^{-\rho s\Delta t} U_{L,t+s\Delta t}\Delta t$.

Definition 2. *(Markov perfect equilibrium in discrete time)* Each governor maximizes the household's life-time utility

$$V^{DD}(H_t, Z_t) = \max_{C_t, L_t, T_t, H_{t+\Delta t}, Z_{t+\Delta t}} U(C_t, L_t)\Delta t + e^{-\rho\Delta t} V^{DD}(H_{t+\Delta t}, Z_{t+\Delta t}), \tag{35}$$

subject to the resource constraint (2), land constraints (3, 4), as well as period-t implementability constraint (34).

The discrete-time Lagrangian with period length Δt is

$$\begin{aligned}
\mathcal{L}^{DD}(H_t, Z_t) = \max_{C_t, L_t, T_t, H_{t+\Delta t}, Z_{t+\Delta t}} & U(C_t, L_t)\Delta t + e^{-\rho\Delta t}\mathcal{L}^{DD}(H_{t+\Delta t}, Z_{t+\Delta t}) \\
& + \mu^{DD}_t [Y_t(Z_t)\Delta t + (1 - \delta\Delta t) Z_t - C_t\Delta t - Z_{t+\Delta t}] \\
& + \lambda^{DD}_t \left[(C_t - Y_t(Z_t) - T_t) U_{C,t}\Delta t + (L_t - H_t) U_{L,t}\Delta t + e^{-\rho\Delta t}\Delta H_{t+\Delta t}\mathcal{U}^{DD}_{L,t+\Delta t}\right] \\
& + \zeta^{DD}_t \Delta t (1 - L_t) + \xi^{DD}_t (1 - H_{t+\Delta t})
\end{aligned} \tag{36}$$

where we attach multiplier μ^{DD}_t to the resource constraint (2), λ^{DD}_t to the time-t implementability condition (34), and ζ^{DD}_t, ξ^{DD}_t to the land constraints. ζ^{DD}_t ($/\xi^{DD}_t$) is equal to zero when L_t ($/H_{t+\Delta t}$) is less than one. Differing from the SB policy analyzed in Section 3.2, here there is a multiplier λ^{DD}_t, which measures the social value of public fund for each time t, instead of a single λ^*.

Further note that $\mathcal{U}^{DD}_{L,t} = \mathcal{U}^{DD}_{L,t}(H_t, Z_t)$ is itself recursive and depends on the optimal policy rule $L^{DD}(H, Z)$, as

$$\mathcal{U}^{DD}_L(H_t, Z_t) \equiv \int_t^\infty e^{-\rho(s-t)} U_L\left(L^{DD}(Z_s, H_s)\right) ds. \tag{37}$$

This is because the time-t planner decides on $H_{t+\Delta t}, Z_{t+\Delta t}$, acknowledging that this choice affects future planners' optimal choices of $L_{t+s\Delta t}$ and thus $U_{L,t+s\Delta t}$.

I characterize the optimality conditions w.r.t. $C_t, L_t, H_{t+\Delta t}, Z_{t+\Delta t}$ and take the limit of $\Delta t \to 0$ to arrive at the optimal equilibrium allocation in continuous time. See Appendix B for detailed derivations.

Proposition 3. (Allocation under double deviation) *The DD allocation* $\left(C_t^{DD}, Z_t^{DD}, L_t^{DD}, H_t^{DD}\right)$ *is completely characterized as follows, given* (Z_0, H_0)*:*

1. *the consumption growth follows*

$$\frac{\dot{C}_t^{DD}}{C_t^{DD}} = \left[Y_t'\left(Z_t^{DD}\right) - (\delta+\rho)\right] - \frac{\lambda_t^{DD}}{\lambda_t^{DD} + \chi_t^{DD}}\left[\chi_t^{DD} Y_t'\left(Z_t^{DD}\right) + \frac{\dot{\chi}_t^{DD}}{\chi_t^{DD}} - \frac{\dot{\lambda}_t^{DD}}{\lambda_t^{DD}} - \chi_t^{DD} C_t^{DD} \dot{H}_t^{DD}\left(\frac{\partial \mathcal{U}_L^{DD}}{\partial Z}\right)_t\right] \tag{38}$$

   with consumption rate $\chi_t^{DD} \equiv \frac{C_t^{DD}}{Y_t^{DD}}$*, subject to the transversality condition and the resource constraint*

$$\dot{Z}_t^{DD} = Y_t\left(Z_t^{DD}\right) - C_t^{DD} - \delta Z_t^{DD}; \tag{39}$$

2. *the land supply satisfies*

$$L_t^{DD} = \min\left\{1, \left[1 + \eta\left(\lambda_t^{DD}\right)\right] H_t^{DD}\right\}, \tag{40}$$

   with excess supply function $\eta(\lambda) \equiv \left(\frac{\lambda\sigma}{1+\lambda} - 1\right)^{-1}$ *which decreases in* λ*;*

3. *the amount of land held in private hands satisfies, when* $H_t^{DD} < 1$*,*

$$\dot{H}_t^{DD} = \frac{\mathcal{U}_{L,t}^{DD}}{\left(\partial \mathcal{U}_L^{DD} / \partial H\right)_t} \frac{\dot{\lambda}_t^{DD}}{\lambda_t^{DD}} \tag{41}$$

4. *when the multiplier* λ_t^{DD} *is positive, it satisfies*

$$\nu\left(L_t^{DD}\right)^{-\sigma}\left(L_t^{DD} - H_t^{DD}\right) + \dot{H}_t^{DD} \mathcal{U}_{L,t}^{DD} = \frac{1}{\chi_t^{DD}} - 1 \tag{42}$$

   whose LHS is increasing in λ_t^{DD} *holding* H_t^{DD} *fixed, when* $L_t^{DD} < 1$*.*

We notice a few interesting differences between the DD allocation in Proposition 3 to the SB allocation in Proposition 2. In terms of consumption growth (38), in addition to the wedge between $Y'\left(Z_t^{DD}\right) - (\delta + \rho)$ and $\dot{C}_t^{DD}/C_t^{DD}$ caused by non-zero λ_t^{DD}, which exists in the SB allocation too, another wedge emerges due to time-varying λ_t^{DD} and H_t^{DD}. In terms of total land supply (40), time-t government always supplies an extra amount relative to H_t^{DD}, which is what is already sold by time t, rather than H_0. And the added supply $\eta\left(\lambda_t^{DD}\right)$ changes over time too, as the social value of public fund λ_t^{DD} varies.

Further, I show in Corollary 2 that when the depreciate rate δ is low, the land market will eventually be saturated ($L_\infty^{DD} = 1$). If the economy grows but the land market is never saturated, assuming that the consumption rate χ_t^{DD} is eventually monotonic and a few other regularity conditions, I establish in Corollary 3 that both H_t^{DD} and L_t^{DD} will strictly increase during convergence.

Corollary 2. *If $\frac{\alpha\delta}{(1-\alpha)\delta+\rho} \leq \frac{\nu}{\sigma}$, the DD steady state features land supply L_∞^{DD} equal to one, i.e. land market saturation.*

Corollary 3. *If the economy grows but land market is never saturated, under certain regularity conditions (i.e., there existing τ such that $\dot{Z}_t^{DD} > 0$, $\left(\frac{\partial \mathcal{U}_L^{DD}}{\partial Z}\right)_t < 0$, $\left(\frac{\partial \mathcal{U}_L^{DD}}{\partial H}\right)_t < 0$ for all $t \geq \tau$, $L_t^{DD} \in \left(\frac{\sigma}{\sigma-1} H_s^{DD}, 1\right)$ for all $t \geq s \geq \tau$, and that χ_t^{DD} is strictly monotonic for all $t \geq \tau$), then it must be that, for $t \geq \tau$, χ_t^{DD} strictly rises, λ_t^{DD} strictly declines, and H_t^{DD}, L_t^{DD} both strictly increase.*

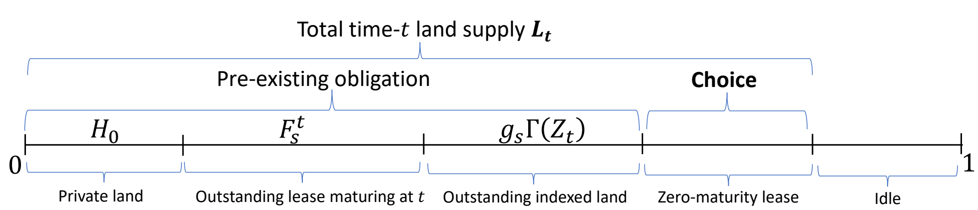
5 Optimal design of land contracts

Last, I show that there exists a better design of land contracts that can help restore the SB allocation, under discretion and incomplete markets. Before deriving the result, we first discuss the time inconsistency issue of the no-borrowing implementation of SB. In order to maintain a balanced budget, the government needs to gradually sell land, i.e., following a specific increasing path H_t^{NB}. Suppose the time-s planner wants to reoptimize, if she has the same λ^* as the time-0 planner, she would supply a total amount of land equal to $[1 + \eta(\lambda^*)] H_s^{NB}$, which will be larger than $L^* = [1 + \eta(\lambda^*)] H_0$. The reason she does not do this under the no-borrowing implementation is that she keeps a promise made by past self; otherwise the past land price would have been lower. In order for her to still choose L^* when reoptimizing, she needs a specific $\lambda^s > \lambda^*$ to ensure $[1 + \eta(\lambda^s)] H_s^{NB} = L^*$. However, if she has such a λ^s that is larger than λ^*, that means she is even poorer relative to FB. In that case, she prefers a lower investment rate $\dot{Z}$ and private return than SB.

This reasoning above clearly illustrates the dilemma between balanced budget and time consistency, and indicates that the specific direction of deviation future governments want is to supply less public capital. When markets are complete, the time-0 governments can leave a specific debt portfolio to future governments which will revaluate adversarially if future governments deviate. Can we achieve the same goal via a land contract, rather than requiring the full maturity structure of debt? The answer is yes, and the optimal land contract penalizes deviation by obliging future governments to give more land to household if the stock of public capital Z_t is low, per unit of land contract outstanding.

We could view land contracts as financial securities that pay land (instead of dollar) to the household by the government. A lease (that is valid for an infinitesimal amount of time at present) is a zero-maturity bond. A sale amounts to a consol that pays one unit of land indefinitely. By no means these are the only possibilities. The government can also sell a future lease F^t, which is a right to use one unit of land for an infinitesimal amount of time at future time t. A future lease F^t is like a zero-coupon bond maturing at time t. A full set of future leases spans all bond-like contracts, including current lease and sale. I denote the outstanding amount at time s of a future lease paying off at time t as F_s^t. Further, the government can sell an indexed land contract that pays a variable amount $\Gamma(Z_t)$ at all t depending on the prevailing level of public capital Z_t. I denote its outstanding amount at time s as g_s. Figure 6 illustrates the land allocation, faced by a time-s planner who chooses time-t allocation for all $t \geq s$. The time-s planner inherits pre-existing obligations including private land H_0, outstanding lease maturing at time t of amount F_s^t, and she is obliged to pay a certain amount of land per unit of outstanding indexed land contract, depending on the time-t stock of public capital she chooses.

Figure 6: Land utilization under multiple land contracts



Proposition 4 suggests that an indexed land contract with a specific dependence $\Gamma(\cdot)$ can be combined with future leases to provide incentives for future governments to continue carrying out the time-0 optimal plan.

Proposition 4. (Optimal design of land contract)

1. *When* $\frac{1}{\chi_\infty^*} - 1 \geq \frac{\nu(L^*)^{1-\sigma}}{\sigma}$ *(or equivalently,* $\frac{\sigma}{\sigma-1}H_\infty^{NB} \leq L^*$*), the SB allocation can be implemented*

under double deviation using future leases and Z-indexed land if and only if

$$\Gamma'(Z_t) = -k\frac{\left(C_t^*\right)^{-1}}{\nu\left(L^*\right)^{-\sigma}}\Delta r_t^*$$

with k being an arbitrary positive constant and $\Delta r_t^* \equiv \left[Y'\left(Z_t^*\right) - (\delta + \rho)\right] - \frac{\dot{C}_t^*}{C_t^*} > 0$ *capturing the wedge between private return and net marginal product of capital at any given time t along the SB convergence path. The amount outstanding at time t of such Z-indexed land* g_s *is increasing over time*

$$g_s = k^{-1}\left(\frac{1}{\lambda^*} - \frac{1}{\lambda^{s*}}\right)$$
$$F_s^t = \frac{L^*}{1+\eta\left(\lambda^{s*}\right)} - H_0 - g_s\Gamma\left(Z_t\right)$$

2. *When* $\frac{1}{\lambda_\infty^*} - 1 < \frac{\nu(L^*)^{1-\sigma}}{\sigma}$*, the same land contract can achieve SB if the government can borrow using at least one bond.*

The two-part obligation consists of an indexed part $g_s\Gamma(Z_t)$ that penalizes future governments supplying less capital (by requiring them to pay out more land since $\Gamma' < 0$), and future leases F_s^t that make up to H_s^{NB} which is the amount of land sale needed to implement SB with no borrowing. This design essentially splits the rights associated with land sale into two parts, so that it can provide incentives without changing the land income. When $\frac{\sigma}{\sigma-1}H_\infty^{NB} \leq L^*$ is violated, it suffices to have one bond for the government to borrow. This is in contrast to the time-consistent implementation of SB allocation discussed in Section 3.3, which requires a full maturity structure of debt to provide incentives. Here, the incentive is provided by the indexed land contract.

Last, we provide some remarks on the feasibility of such an indexed land contract. The required legal protection of future leases and indexed land contract is the same as outright sale, in that they are all subject to expropriation. In order for this contract design to work, the indexed land contract has to be honored by future governments. This is a fair requirement, as we note that in order to implement the SB allocation under complete markets à la Lucas and Stokey (1983) and Debortoli, Nunes and Yared (2021), debt contracts issued by the current government have to be acknowledged by future governments too. Further, we note that, while a land contract indexed on capital stock Z_t can help resume the SB allocation, there exist other indexed contracts based on other variables that could achieve the same goal. The advantage of an indexed land contract based on capital stock is that Z_t is easily verifiable, since public investment is an important item in national accounts, and is directly controlled by the government. The timeless dependence $\Gamma(\cdot)$ is easy to enforce too: since it is differentiable, there is no need to argue about a small amount of measurement error in court.

6 Conclusion

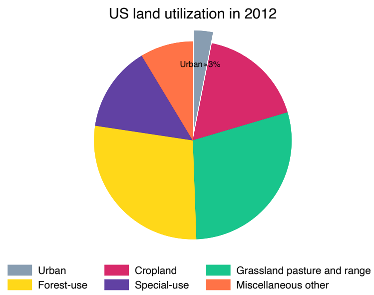
Better understanding land finance can help improve policies concerning billions of people in the developing world. This paper makes three contributions regarding the mechanism and design of land finance.. First, I provide a model to understand land finance and show that the optimal policy should front-load public investment while supplying a constant amount of land over time. Second, I offer an explanation for the observed increase in land supply in practice, based on the combination of discretion and financial constraints. Lastly, I propose a realistic design of land contracts that links land supply to the provision of public goods to restore the optimal allocation.

This paper focuses on the efficiency aspect of land finance in a deterministic environment. It presents a neoclassical growth model augmented with the role of land finance, which can serve as a starting point for other models. First, future research may fruitfully explore stochastic growth, especially the pricing of land in incomplete markets and its implications for land contract design. Second, with a neoclassical growth model at hand, future researchers may also build business cycle models or New Keynesian models to study cyclical fluctuations. If the government is subject to collateral constraints, the use of land finance may create a financial accelerator (Bernanke, Gertler and Gilchrist, 1999) that goes through the public sector rather than the private sector. Third, while this paper studies efficiency in a representative agent model, housing and land play crucial roles in driving wealth inequality across countries (Bonnet et al., 2021). In light of that, how should the policymaker design land finance policies to take into account distributional impact? Last but not least, this paper addresses a monopoly problem. In practice, land is supplied by many local governments instead of a consolidated government. It is of interest to explore how imperfect competition and migration matter for land finance.

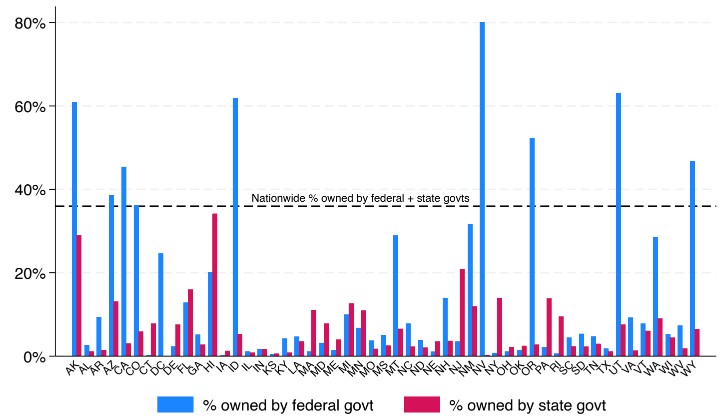
A Additional figures

Figure A1: Land utilization and ownership in the US

(a) Land utilization in 2012



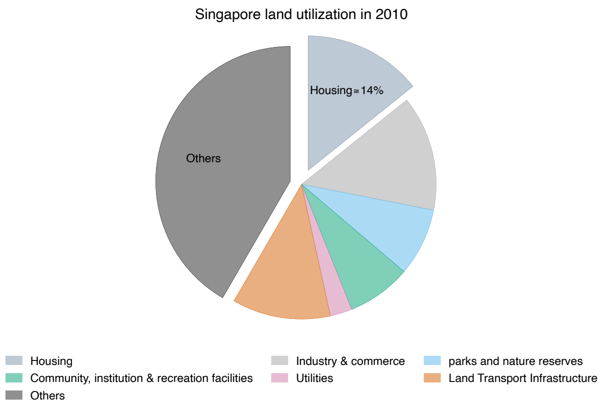
(b) Public land ownership in 2018



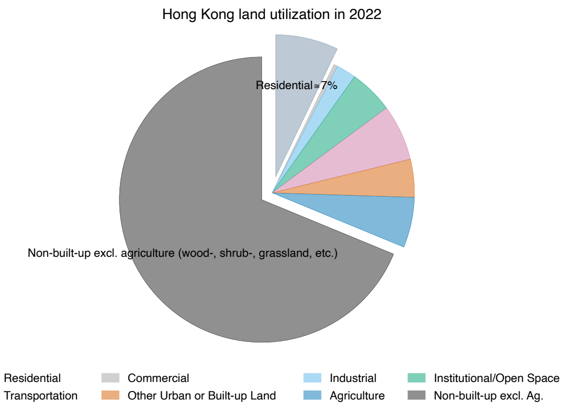
Notes: Data of public land ownership are from Nelson (2018) and Vincent and Hanson (2020).

Figure A2: Land utilization in Singapore, Hong Kong and mainland China

(a) Singapore in 2010



(b) Hong Kong in 2022



(c) Mainland China in 2022

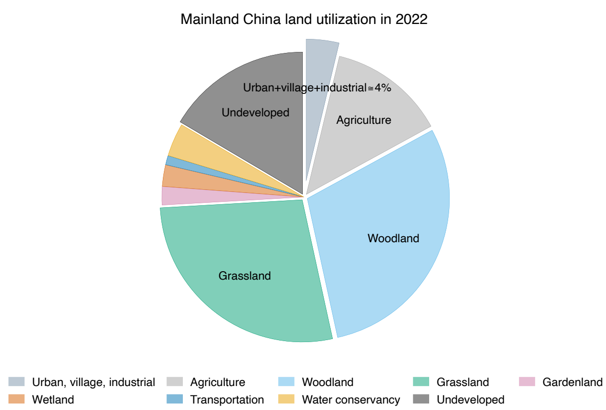
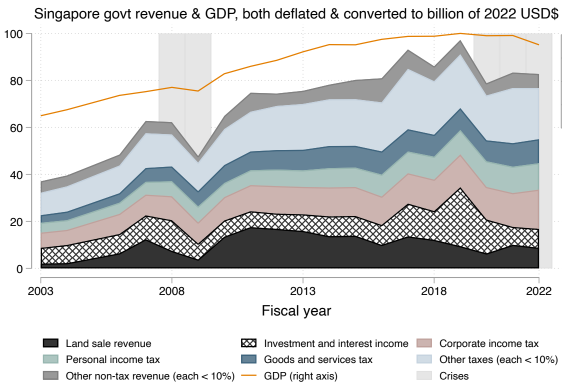
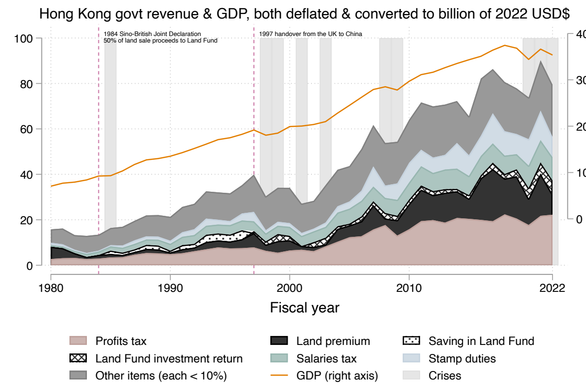


Figure A3: Land sale in government revenue of Asian growth miracles

(a) Singapore, 2003-2022



(b) Hong Kong, 1980-2022



(c) Mainland China, 2001-2022

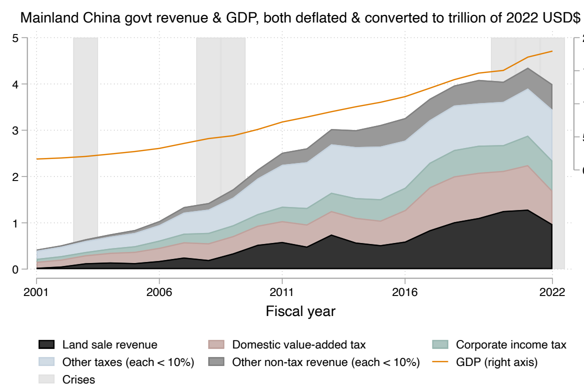
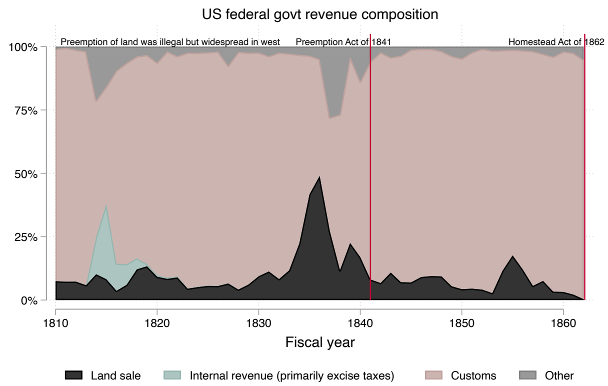
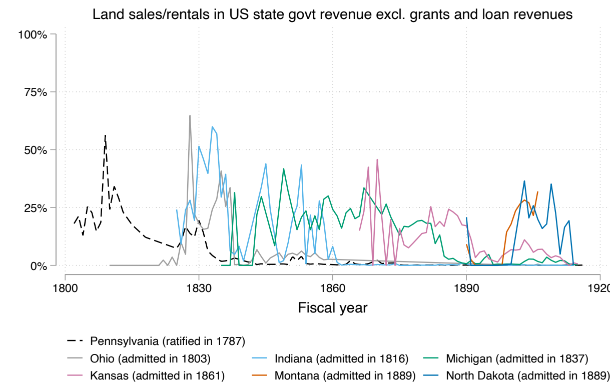


Figure A4: Land sale in US and Hong Kong government revenue in 19th century

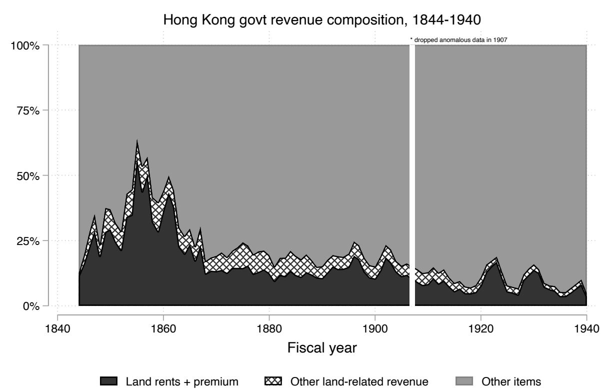
(a) US federal government



(b) US state governments



(c) Hong Kong, 1844-1940



Notes: Data of US federal and state government revenues are from Wallis (2006) and Sylla, Legler and Wallis (1993) respectively. Explain US practice, and data of Hong Kong are from HKIMR Hong Kong Economic History Database.

B Proofs of main results

Proof of Proposition 1.

$$\begin{aligned}
\max \mathcal{L}^{FB} &= \int_0^\infty \mathrm{e}^{-\rho t} U(C_t, L_t)\,\mathrm{d}t + \int_0^\infty \mu_t \mathrm{e}^{-\rho t}\left[Y_t(Z_t) - C_t - \delta Z_t - \dot{Z}_t\right]\mathrm{d}t \\
&= \int_0^\infty \mathrm{e}^{-\rho t} U(C_t, L_t)\,\mathrm{d}t + \int_0^\infty \mathrm{e}^{-\rho t}\left\{\mu_t\left[Y_t(Z_t) - C_t - (\delta+\rho) Z_t\right] + \dot{\mu}_t Z_t\right\}\mathrm{d}t + \mu_0 Z_0
\end{aligned}$$

The optimality conditions are

$$\begin{aligned}
0 &= \mathrm{e}^{\rho t}\frac{\partial \mathcal{L}^{FB}}{\partial C_t} = U_{C,t} - \mu_t \\
0 &= \mathrm{e}^{\rho t}\frac{\partial \mathcal{L}^{FB}}{\partial Z_t} = \dot{\mu}_t + \left[Y_t' - (\delta+\rho)\right]\mu_t
\end{aligned}$$

Thus, the consumption growth rate is

$$\frac{\dot{C}_t}{C_t} = Y_t' - (\delta + \rho)$$

□

Proof of Proposition 2.

Consumption growth and fiscal balance. Use (22) to get

$$\mu_t^* = (1 + \lambda^* Y_t / C_t)\, C_t^{-1} = \left(1 + \frac{\lambda^*}{\chi_t}\right) C_t^{-1}$$

Plug this into (24) and arrive at

$$0 = \left(1 + \frac{\lambda^*}{\chi_t}\right) C_t^{-1}\left[Y_t' - (\delta+\rho)\right] - \left(1 + \frac{\lambda^*}{\chi_t}\right) C_t^{-2}\dot{C}_t - \frac{\lambda^*}{(\chi_t)^2} C_t^{-1}\dot{\chi}_t - \lambda^* Y_t' C_t^{-1}$$

which simplifies to

$$\frac{\dot{C}_t^*}{C_t^*} = \left[Y_t'(Z_t^*) - (\delta+\rho)\right] - \frac{\lambda^*}{\lambda^* + \chi_t^*}\left[\chi_t^* Y_t'(Z_t^*) + \frac{\dot{\chi}_t^*}{\chi_t^*}\right]$$

For any (Z_0, H_0), the multiplier λ^* satisfies the implementability condition (20). We write (20) as equalizing fiscal income and expenditure in unit of utility

$$\underbrace{\int_0^\infty e^{-\rho t} (L_t - H_0)\, U_{L,t} dt}_{\equiv \mathcal{I}^*} = \underbrace{\int_0^\infty e^{-\rho t} (Y_t - C_t)\, U_{C,t} dt}_{\equiv \mathcal{E}^*} \tag{B1}$$

The present-value util-unit sum of fiscal income $\mathcal{I}^*$ equals

$$\mathcal{I}^* = \int_0^\infty e^{-\rho t} \eta H_0 U_L ((1+\eta) H_0)\, dt = \frac{\nu H_0^{1-\sigma}}{\rho} \frac{\eta}{(1+\eta)^\sigma}$$

which only depends on (H_0, λ^*), i.e., $\mathcal{I}^* = \mathcal{I}^*(H_0, \lambda^*)$. It satisfies $\frac{\partial \mathcal{I}^*}{\partial H_0} < 0$ and

$$\frac{\partial \mathcal{I}^*}{\partial \lambda^*} = \frac{\nu H_0^{1-\sigma}}{\rho} \frac{1 - (\sigma - 1)\eta}{(1+\eta)^{\sigma+1}} \frac{d\eta}{d\lambda^*} = -\frac{\nu H_0^{1-\sigma}}{\rho} (1+\eta)^{-(\sigma+1)} \frac{\eta\sigma}{1+\lambda^*} \frac{d\eta}{d\lambda^*} > 0$$

Intuitively, the larger the need for fiscal income (higher λ^*), the larger the fiscal income ($\mathcal{I}^*$). $\mathcal{I}^*$ is independent of Z_0 except through its dependence on λ^*.

The present-value util-unit sum of fiscal expenditure $\mathcal{E}^*$ equals

$$\mathcal{E}^* = \int_0^\infty e^{-\rho t} \left(\frac{1}{\chi_t} - 1 \right) dt$$

which implicitly depends on Z_0 and λ^* through the ODE system (2, **??**), i.e., $\mathcal{E}^* = \mathcal{E}^*(Z_0, \lambda^*)$.

Phase diagram in (Z, C) space. The $\dot{Z} = 0$ locus is from (28) as

$$C_t^* = Y(Z_t^*) - \delta Z_t^*$$

which is concave and positive between 0 and $\left(\frac{A}{\delta}\right)^{\frac{1}{1-\alpha}}$, the same as in the neoclassical growth model. Its apex value is $C^a = \left(\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right)\left(\frac{A}{\delta^\alpha}\right)^{\frac{1}{1-\alpha}}$ at $Z^a = \left(\frac{\alpha A}{\delta}\right)^{\frac{1}{1-\alpha}}$.

To arrive at the $\dot{C} = 0$ locus, note that

$$\frac{\dot{\chi}_t^*}{\chi_t^*} = \frac{\dot{C}_t^*}{C_t^*} - \frac{\dot{Y}_t^*}{Y_t^*} = \frac{\dot{C}_t^*}{C_t^*} - \alpha \frac{\dot{Z}_t^*}{Z_t^*} = \frac{\dot{C}_t^*}{C_t^*} - \alpha \frac{Y_t^* - C_t^* - \delta Z_t^*}{Z_t^*}$$

where we have plugged in the production function and (28). Then, using (27) and setting

$\dot{C}_t^* = 0$, we get

$$
\begin{aligned}
0 &= Y'\left(Z_t^*\right) - (\delta+\rho) - \frac{\lambda^*}{\lambda^* + C_t^*/Y\left(Z_t^*\right)}\left[\frac{C_t^*}{Y\left(Z_t^*\right)}Y'\left(Z_t^*\right) - \alpha\frac{Y\left(Z_t^*\right) - C_t^* - \delta Z_t^*}{Z_t^*}\right] \\
&= Y'\left(Z_t^*\right) - (\delta+\rho) + \frac{\alpha\lambda^*}{\lambda^* + C_t^*/Y\left(Z_t^*\right)}\frac{Y\left(Z_t^*\right) - 2C_t^* - \delta Z_t^*}{Z_t^*} \qquad \text{(B2)}
\end{aligned}
$$

In the case with $\lambda^* = 0$, this locus is $0 = Y'\left(Z_t^*\right) - (\delta+\rho)$ which specifies a constant $Z_t^* \equiv Z_\infty^{FB} = \left(\frac{\alpha A}{\delta+\rho}\right)^{\frac{1}{1-\alpha}}$ (a vertical line in (Z, C) space) as in the neoclassical model. Its intersection with the $\dot{Z} = 0$ locus is $C_\infty^{FB} = Y\left(Z_\infty^{FB}\right) - \delta Z_\infty^{FB}$. For any $\lambda^* > 0$, the $\dot{C} = 0$ locus is no longer a vertical line at Z_∞^{FB}, but it will cross that vertical line once at exactly $\left(Z_\infty^{FB}, C_\infty^{FB}/2\right)$.[14] We will prove in Corollary 1 that the steady state C_∞^*, Z_∞^* both decrease in λ^*, suggesting that the intersection of $\dot{Z} = 0$ locus and $\dot{C} = 0$ locus under $\lambda^* > 0$ is to the left of Z_∞^{FB}. That is, the $\dot{C} = 0$ locus is rotated counterclockwise around $\left(Z_\infty^{FB}, C_\infty^{FB}/2\right)$ for $\lambda^* > 0$.

Monotonicity of consumption rate χ_t. Omit the time subscript and $*$ superscript here for simplicity. We have

$$
\begin{aligned}
g^\chi \equiv \frac{\dot{\chi}}{\chi} &= \frac{\dot{C}}{C} - \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} - \alpha\frac{\dot{Z}}{Z} = \frac{\dot{C}}{C} - \alpha\frac{Y - C - \delta Z}{Z} \\
&= \left[Y' - (\delta+\rho)\right] - \frac{\lambda}{1+\frac{\lambda}{\chi}}\left[\frac{\dot{\chi}}{\chi^2} + Y'\right] - Y'(1-\chi) + \alpha\delta \\
&= Y'\chi - \left[(1-\alpha)\delta + \rho\right] - \frac{\lambda}{1+\frac{\lambda}{\chi}}\left(\frac{\dot{\chi}}{\chi^2} + Y'\right) \\
&= Y'\frac{\chi^2}{\chi+\lambda} - \left[(1-\alpha)\delta + \rho\right] - \frac{\lambda}{\chi+\lambda}g^\chi \\
\frac{\chi+2\lambda}{\chi+\lambda}g^\chi &= Y'\frac{\chi^2}{\chi+\lambda} - \left[(1-\alpha)\delta + \rho\right] \qquad \text{(B3)}
\end{aligned}
$$

[14] To be precise, there is only one crossing in the $\dot{Z} > 0$ subspace ($C \leq Y(Z) - \delta Z$). Outside that, there is another uninteresting crossing at $C \to \infty$ which corresponds to infinitely fast depreciation of Z.

Let $\gamma \equiv \frac{\chi+2\lambda}{\chi+\lambda} g^{\chi}$ which has the same sign as g^{χ}. We have

$$\gamma = Y' \frac{\chi^2}{\chi+\lambda} - [(1-\alpha)\delta + \rho]$$
$$\dot{\gamma} = Y'' \dot{Z} \frac{\chi^2}{\chi+\lambda} + Y' \frac{(\chi+2\lambda)\chi^2}{(\chi+\lambda)^2} g^{\chi}$$
$$= Y'' \dot{Z} \frac{\chi^2}{\chi+\lambda} + Y' \frac{\chi^2}{\chi+\lambda} \gamma$$

If $\gamma \leq 0$ for some t, then $\dot{\gamma} < 0$ since $Y'' < 0, \dot{Z} > 0, Y' > 0$ at that time, as Z monotonically increases when starting from the left of Z_{∞}^{*} in the third quadrant on the phase diagram. It suggests that $\gamma < 0$ holds forever and γ keeps decreasing, a result inconsistent with convergence. Thus we must have $\gamma > 0$ for all t, which means $g^{\chi} > 0$ and thus $\dot{\chi} > 0$.□

Proof of Corollary 1. At the long-run steady state, (27) gives

$$\left(1 - \frac{\lambda^{*}}{1 + \frac{\lambda^{*}}{\chi_{\infty}^{*}}}\right) Y_{\infty}' = \delta + \rho \tag{B4}$$

Case with $\delta = 0$. If $\delta = 0$, then $\chi_{\infty}^{*} = 1$ from the resource constraint and thus (B4) reduces to $Y_{\infty}' = (1+\lambda^{*})\rho$. Hence

$$Z_{\infty}^{*} = (Y')^{-1}\left((1+\lambda^{*})\rho\right) = \left[\frac{\alpha}{(1+\lambda^{*})\rho}\right]^{\frac{1}{1-\alpha}}$$

and $C_{\infty}^{*} = Y_{\infty}^{*} = A(Z_{\infty}^{*})^{\alpha}$. Obviously, both decrease in λ^{*}.

Case with $\delta > 0$. If $\delta > 0$, the resource constraint implies that $Z_{\infty}^{*} = \frac{Y_{\infty}^{*} - C_{\infty}^{*}}{\delta}$ and thus the marginal product is

$$Y_{\infty}' = \frac{\alpha Y_{\infty}^{*}}{Z_{\infty}^{*}} = \frac{\alpha\delta}{1-\chi_{\infty}^{*}} \tag{B5}$$

Plugging this in (B4) gives

$$\left(1 - \frac{\lambda^{*}}{1 + \frac{\lambda^{*}}{\chi_{\infty}^{*}}}\right) \frac{\alpha\delta}{1-\chi_{\infty}^{*}} = \delta + \rho$$

$$\left(1+\frac{\chi^*_\infty}{1-\chi^*_\infty-\lambda^*+\frac{\lambda^*}{\chi^*_\infty}}\right)\alpha\delta=\delta+\rho$$

$$\frac{\chi^*_\infty}{1-\chi^*_\infty-\lambda^*+\frac{\lambda^*}{\chi^*_\infty}}=\frac{\delta\left(1-\alpha\right)+\rho}{\alpha\delta}$$

It is a quadratic equation in χ^*

$$(\delta+\rho)\left(\chi^*_\infty\right)^2-(1-\lambda^*)\left[\delta\left(1-\alpha\right)+\rho\right]\chi^*_\infty-\lambda^*\left[\delta\left(1-\alpha\right)+\rho\right]=0$$

which gives two solutions

$$\begin{aligned}\chi^*_\infty&=\frac{(1-\lambda^*)\left[\delta\left(1-\alpha\right)+\rho\right]\pm\sqrt{(1-\lambda^*)^2\left[\delta\left(1-\alpha\right)+\rho\right]^2+4\left(\delta+\rho\right)\lambda^*\left[\delta\left(1-\alpha\right)+\rho\right]}}{2\left(\delta+\rho\right)}\\&=\frac{(1-\lambda^*)\pm\sqrt{(1-\lambda^*)^2+4\frac{\delta+\rho}{\delta\left(1-\alpha\right)+\rho}\lambda^*}}{2}\frac{\delta\left(1-\alpha\right)+\rho}{\delta+\rho}\\&=\frac{(1-\lambda^*)\pm\sqrt{(1+\lambda^*)^2+4\frac{\delta\alpha}{\delta\left(1-\alpha\right)+\rho}\lambda^*}}{2}\frac{\delta\left(1-\alpha\right)+\rho}{\delta+\rho}\end{aligned}$$

We take the solution with plus sign since consumption can only be positive.

χ^*_∞ increases in λ^* since

$$\frac{\mathrm{d}\chi^*_\infty}{\mathrm{d}\lambda^*}\propto-1+\frac{(1+\lambda^*)+2\frac{\delta\alpha}{\delta(1-\alpha)+\rho}}{\sqrt{(1+\lambda^*)^2+4\frac{\delta\alpha}{\delta\left(1-\alpha\right)+\rho}\lambda^*}}>0$$

(B5) gives infrastructure

$$Z^*_\infty=(Y')^{-1}\left(\frac{\alpha\delta}{1-\chi^*_\infty}\right)=\left[\frac{A\left(1-\chi^*_\infty\right)}{\delta}\right]^{\frac{1}{1-\alpha}}$$

which decreases in χ^*_∞ and thus increases in λ^*.

Further, the consumption is

$$C_\infty^* = \chi_\infty^* Y_\infty^* = A\chi_\infty^* \left[\frac{A\left(1-\chi_\infty^*\right)}{\delta}\right]^{\frac{\alpha}{1-\alpha}}$$

To show that it is increasing in λ^*, it suffices to show that $\chi_\infty^*\left(1-\chi_\infty^*\right)^{\frac{\alpha}{1-\alpha}}$ decreases in χ_∞^*. We check

$$\frac{\mathrm{d}\log\left[\chi_\infty^*\left(1-\chi_\infty^*\right)^{\frac{\alpha}{1-\alpha}}\right]}{\mathrm{d}\chi_\infty^*} = \frac{1}{\chi_\infty^*} - \frac{\alpha}{1-\alpha}\frac{1}{1-\chi_\infty^*} = \frac{1-\alpha-\chi_\infty^*}{\left(1-\alpha\right)\left(1-\chi_\infty^*\right)\chi_\infty^*}$$

which is negative since $\chi_\infty^* > \frac{\delta(1-\alpha)+\rho}{\delta+\rho} > 1-\alpha$.□

Proof of Proposition 3. For simplicity I suppress the superscript DD. The FOCs w.r.t. C_t, L_t of (36) are

$$\begin{aligned} 0 &= U_{C,t}\Delta t - \lambda_t \Delta t Y_t U_{CC,t} - \mu_t \Delta t \\ 0 &= \left(1+\lambda_t\right) U_{L,t}\Delta t + \lambda_t\left(L_t - H_t\right) U_{LL,t}\Delta t - \zeta_t \Delta t \end{aligned}$$

and the limits of $\Delta \to 0$ are, assuming an interior solution $L_t < 1$,

$$\begin{aligned} 0 &= U_{C,t} - \lambda_t Y_t U_{CC,t} - \mu_t \\ 0 &= \left(1+\lambda_t\right) U_{L,t} + \lambda_t\left(L_t - H_t\right) U_{LL,t} - \zeta_t \end{aligned}$$

The envelope conditions from (36) are

$$\begin{aligned} \mathcal{L}_H^{DD}\left(H_t, Z_t\right) &= -\lambda_t\left(U_{L,t}\Delta t + \mathrm{e}^{-\rho\Delta t}\mathcal{U}_{L,t+\Delta t}\right) = -\lambda_t \mathcal{U}_{L,t} \\ \mathcal{L}_Z^{DD}\left(H_t, Z_t\right) &= \mu_t\left[Y_t'\Delta t + \left(1-\delta\Delta t\right)\right] - \lambda_t Y_t' U_{C,t}\Delta t \end{aligned}$$

The optimality conditions w.r.t. $H_{t+\Delta t}, Z_{t+\Delta t}$ of (36) are

$$\begin{aligned} 0 &= \mathrm{e}^{-\rho\Delta t}\mathcal{L}_{H,t+\Delta t}^{DD} + \lambda_t \mathrm{e}^{-\rho\Delta t}\mathcal{U}_{L,t+\Delta t} + \mathrm{e}^{-\rho\Delta t}\lambda_t \Delta H_{t+\Delta t}\frac{\partial \mathcal{U}_{L,t+\Delta t}}{\partial H_{t+\Delta t}} - \xi_t \\ &= \mathrm{e}^{-\rho\Delta t}\left[\left(-\lambda_{t+\Delta t}+\lambda_t\right)\mathcal{U}_{L,t+\Delta t} + \lambda_t \Delta H_{t+\Delta t}\frac{\partial \mathcal{U}_{L,t+\Delta t}}{\partial H_{t+\Delta t}}\right] - \xi_t \end{aligned} \tag{B6}$$

$$
\begin{aligned}
0 &= \mathrm{e}^{-\rho\Delta t}\mathcal{L}^{DD}_{Z,t+\Delta t} - \mu_t + \mathrm{e}^{-\rho\Delta t}\lambda_t \Delta H_{t+\Delta t}\frac{\partial \mathcal{U}_{L,t+\Delta t}}{\partial Z_{t+\Delta t}} \\
&= \mathrm{e}^{-\rho\Delta t}\left\{\mu_{t+\Delta t}\left[Y'_{t+\Delta t}\Delta t + (1-\delta\Delta t)\right] - \lambda_{t+\Delta t}Y'_{t+\Delta t}U_{C,t+\Delta t}\Delta t + \lambda_t \Delta H_{t+\Delta t}\frac{\partial \mathcal{U}_{L,t+\Delta t}}{\partial Z_{t+\Delta t}}\right\} - \mu_t
\end{aligned} \tag{B7}
$$

In simplifying last two expressions, we have plugged in the envelope conditions.

When $H_{t+\Delta t} < 1$ and thus $\xi_t = 0$, the limit of $\Delta t \to 0$ of (B6) is

$$
0 = -\frac{\dot{\lambda}_t}{\lambda_t}\mathcal{U}_{L,t} + \dot{H}_t\left(\frac{\partial \mathcal{U}_L}{\partial H}\right)_t
$$

The limit of $\Delta t \to 0$ of $\mathrm{e}^{\rho\Delta t}\times$(B7) is

$$
\begin{aligned}
0 &= \mu_{t+\Delta t}\left[Y'_{t+\Delta t}\Delta t + (1-\delta\Delta t)\right] - \lambda_{t+\Delta t}Y'_{t+\Delta t}U_{C,t+\Delta t}\Delta t + \lambda_t \Delta H_{t+\Delta t}\frac{\partial \mathcal{U}_{L,t+\Delta t}}{\partial Z_{t+\Delta t}} - \mu_t \mathrm{e}^{\rho\Delta t} \\
&= \mu_{t+\Delta t}\Delta t\left(Y'_{t+\Delta t} - \delta\right) + \dot{\mu}_t\Delta t - \rho\mu_t\Delta t - \lambda_{t+\Delta t}Y'_{t+\Delta t}U_{C,t+\Delta t}\Delta t + \lambda_t\dot{H}_t\Delta t\frac{\partial \mathcal{U}_{L,t+\Delta t}}{\partial Z_{t+\Delta t}} \\
0 &= \left(U_{C,t} - \lambda_t Y_t U_{CC,t}\right)\left(Y'_t - \rho - \delta\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(U_{C,t} - \lambda_t Y_t U_{CC,t}\right) - \lambda_t Y'_t U_{C,t} + \lambda_t \dot{H}_t\left(\frac{\partial \mathcal{U}_L}{\partial Z}\right)_t \\
\frac{\dot{C}_t}{C_t} &= \left[Y'_t(Z_t) - (\delta+\rho)\right] - \frac{\lambda_t}{\lambda_t + \chi_t}\left[\chi_t Y'_t(Z_t) + \frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\lambda}_t}{\lambda_t} - \chi_t C_t \dot{H}_t\left(\frac{\partial \mathcal{U}_L}{\partial Z}\right)_t\right] \\
&= \left[Y'_t(Z_t) - (\delta+\rho)\right] - \frac{\lambda_t}{\lambda_t + \chi_t}\left[\chi_t Y'_t(Z_t) + \frac{\dot{\chi}_t}{\chi_t} - \frac{\dot{\lambda}_t}{\lambda_t} - \chi_t C_t \frac{\mathcal{U}_{L,t}}{\left(\partial \mathcal{U}_L/\partial H\right)_t}\frac{\dot{\lambda}_t}{\lambda_t}\left(\frac{\partial \mathcal{U}_L}{\partial Z}\right)_t\right] \\
&= \left[Y'_t(Z_t) - (\delta+\rho)\right] - \frac{\lambda_t}{\lambda_t + \chi_t}\left[\chi_t Y'_t(Z_t) + \frac{\dot{\chi}_t}{\chi_t} - \left(1 + \chi_t C_t \frac{\mathcal{U}_{L,t}\left(\partial \mathcal{U}_L/\partial Z\right)_t}{\left(\partial \mathcal{U}_L/\partial H\right)_t}\right)\frac{\dot{\lambda}_t}{\lambda_t}\right]
\end{aligned}
$$

Last, the limit under $\Delta t \to 0$ of the time-t implementability condition is

$$
\begin{aligned}
0 &= \left(C_t - Y_t(Z_t)\right)U_{C,t}\Delta t + \left(L_t - H_t\right)U_{L,t}\Delta t + \mathrm{e}^{-\rho\Delta t}\Delta H_{t+\Delta t}\mathcal{U}_{L,t+\Delta t} \\
0 &= \left(C_t - Y_t(Z_t)\right)U_{C,t} + \left(L_t - H_t\right)U_{L,t} + \dot{H}_t\mathcal{U}_{L,t}
\end{aligned}
$$

which simplifies to

$$
\nu L_t^{-\sigma}\left(L_t - H_t\right) + \dot{H}_t\mathcal{U}_{L,t} = \frac{1}{\chi_t} - 1
$$

□

Proof of Corollary 2. For simplicity I suppress the superscript DD when there is no ambiguity. If the steady state $L_\infty < 1$, then we must have $\lambda_\infty > 0$ and $L_\infty = \left[1 + \eta(\lambda_\infty)\right]H_\infty$

from (40). The fiscal budget (42) implies that

$$\nu\left(L_{\infty}\right)^{1-\sigma}\frac{\eta\left(\lambda_{\infty}\right)}{1+\eta\left(\lambda_{\infty}\right)}=\frac{1}{\chi_{\infty}}-1 \tag{B8}$$

Now we seek a sufficient condition under which (B8) can never hold. First, note that the DD steady state χ_{∞} follows the same expression as in Corollary 1 if we replace λ^{*} by λ_{∞}^{DD}. Thus the property in Corollary 1 implies that

$$\chi_{\infty}^{DD}>\chi_{\infty}^{FB}=\frac{\delta\left(1-\alpha\right)+\rho}{\delta+\rho}$$

and hence

$$\frac{1}{\chi_{\infty}^{DD}}-1<\frac{\alpha\delta}{\left(1-\alpha\right)\delta+\rho}$$

Second, the LHS (B8) decreases in both L_{∞} and λ_{∞}, and thus

$$\nu\left(L_{\infty}\right)^{1-\sigma}\frac{\eta\left(\lambda_{\infty}\right)}{1+\eta\left(\lambda_{\infty}\right)}>\nu\frac{\eta\left(\infty\right)}{1+\eta\left(\infty\right)}=\frac{\nu}{\sigma}$$

There, (B8) never holds if

$$\frac{\alpha\delta}{\left(1-\alpha\right)\delta+\rho}\leq\frac{\nu}{\sigma}$$

That means L_{∞} must be equal to one.□

Proof of Corollary 3. Though the Corollary is stated for a generic τ, note that we are studying a Markov Perfect Equilibrium and thus without loss of generality we can start our analysis at time τ, i.e. relabeling $t-\tau$ as t and focusing on later periods.

For simplicity I suppress the superscript DD. Assume that $\dot{Z}_{t}>0$, $\left(\frac{\partial\mathcal{U}_{L}}{\partial Z}\right)_{t}<0$ and $\left(\frac{\partial\mathcal{U}_{L}}{\partial H}\right)_{t}<0$ for all t. Assume that $L_{t}\in\left(\frac{\sigma}{\sigma-1}H_{s},1\right)$ for all $t\geq s\geq0$, and thus (40) takes the interior solution $L_{t}=\left[1+\eta\left(\lambda_{t}\right)\right]H_{t}$. Further, assume that χ_{t} is strictly monotonic and λ_{t} is twice differentiable.

Step 1: If $\dot{\lambda}$ has the same sign as $\dot{\chi}$ at some time s, they must have the same sign for all $t\geq s$. For any τ such that $\dot{\lambda}_{\tau}=0$, plug (40, 41) into (42) and evaluate its time derivative at τ,

$$\frac{\left(\mathcal{U}_{L,\tau}\right)^{2}}{\left(\partial\mathcal{U}_{L}/\partial H\right)_{\tau}}\frac{\ddot{\lambda}_{\tau}}{\lambda_{\tau}}=-\frac{\dot{\chi}_{\tau}}{\left(\chi_{\tau}\right)^{2}}$$

As $\left(\frac{\partial \mathcal{U}_L}{\partial H}\right)_\tau < 0$, $\ddot{\lambda}_\tau$ must have the same sign as $\dot{\chi}_\tau$. Thus $\dot{\lambda}$ crosses zero at most once, and after that point, $\dot{\lambda}$ must have the same sign as $\dot{\chi}$.

Step 2: $\dot{\lambda}_t$ must have the opposite sign from $\dot{\chi}_t$ for all t. From step 1, if $\dot{\lambda}_t$ ever has the same sign as $\dot{\chi}_t$ at some point, there must exists $s \geq 0$ such that $\dot{\lambda}_t$ has the same sign as $\dot{\chi}_t$ for all $t \geq s$. I show that this cannot happen. Integrate $\mathrm{e}^{-\rho(s-t)} \times$(42) from such a time s to infinity and get the LHS as

$$\begin{aligned}
\mathcal{I}^s &\equiv \int_s^\infty \mathrm{e}^{-\rho(t-s)} \left[U_{L,t} (L_t - H_t) + \mathcal{U}_{L,t} \dot{H}_t \right] \mathrm{d}t \\
&= \int_s^\infty \mathrm{e}^{-\rho(t-s)} U_{L,t} (L_t - H_t) \, \mathrm{d}t + \int_s^\infty \mathrm{e}^{-\rho(t-s)} \mathcal{U}_{L,t} \mathrm{d}H_t \\
&= \int_s^\infty \mathrm{e}^{-\rho(t-s)} U_{L,t} (L_t - H_t) \, \mathrm{d}t - \mathcal{U}_{L,s} H_s - \int_s^\infty H_t \mathrm{d} \left(\mathrm{e}^{-\rho(t-s)} \mathcal{U}_{L,t} \right) \\
&= \int_s^\infty \mathrm{e}^{-\rho(t-s)} U_{L,t} (L_t - H_s) \, \mathrm{d}t
\end{aligned}$$

The intuition for this transformation is that future land sale is fairly priced too, the same as why H_t does not appear in the time-0 implementability condition (20) under SB allocation. Now we plug in the optimal land supply (40) and write

$$\mathcal{I}^s = \int_s^\infty \mathrm{e}^{-\rho(t-s)} \iota_{t,s} \mathrm{d}t$$

with $\iota_{t,s} = \nu \left[(1+\eta_t)^{1-\sigma} H_t^{1-\sigma} - (1+\eta_t)^{-\sigma} H_t^{-\sigma} H_s \right], \eta_t = \eta(\lambda_t)$. That implies that $\rho \mathcal{I}^s$ is a weighted average of $\iota_{t,s}$ over $t \geq s$. If $\iota_{t,s}$ is strictly decreasing (/increasing) for all $t \geq s$, we would have $\iota_{s,s}$ to be strictly larger (/smaller) than $\rho \mathcal{I}^s$. We take the derivative of $\iota_{t,s}$ w.r.t. t to get

$$\begin{aligned}
\frac{\mathrm{d}\iota_{t,s}}{\mathrm{d}t} &= \nu (1+\eta_t)^{-\sigma} H_t^{-\sigma} \left[(1-\sigma) \eta_t' \dot{\lambda}_t H_t + (1-\sigma)(1+\eta_t) \dot{H}_t + \sigma \frac{\eta_t' \dot{\lambda}_t}{1+\eta_t} H_s + \sigma \frac{\dot{H}_t}{H_t} H_s \right] \\
&= -\nu (\sigma - 1)(1+\eta_t)^{-\sigma} H_t^{-\sigma-1} \left[(1+\eta_t) H_t - \frac{\sigma}{\sigma-1} H_s \right] \left(\dot{H}_t + \frac{\eta_t' \dot{\lambda}_t}{1+\eta_t} H_t \right) \\
&= -\nu (\sigma - 1)(1+\eta_t)^{-\sigma} H_t^{-\sigma-1} \left(L_t - \frac{\sigma}{\sigma-1} H_s \right) \left(\frac{\mathcal{U}_{L,t}}{(\partial \mathcal{U}_L / \partial H)_t} \frac{1}{\lambda_t} + \frac{\eta_t'}{1+\eta_t} H_t \right) \dot{\lambda}_t
\end{aligned}$$

of which the last step uses (40) and (41). Since $L_t > \frac{\sigma}{\sigma-1}H_s$, $(\partial\mathcal{U}_L/\partial H)_t < 0$, $\eta' < 0$, and $\sigma > 1$, we observe that $\frac{\mathrm{d}\iota_{t,s}}{\mathrm{d}t}$ has the same sign as $\dot{\lambda}_t$. Thus if λ_t is strictly decreasing (/increasing) for all $t \geq s$, we would have $\iota_{s,s}$ to be strictly larger (/smaller) than $\rho\mathcal{I}^s$.

Note that $\mathcal{E}^s \equiv \int_s^\infty \mathrm{e}^{-\rho(t-s)}\left(\frac{1}{\chi_t}-1\right)\mathrm{d}t$ also implies that $\rho\mathcal{E}^s$ is a weighted average of $\frac{1}{\chi_t}-1$ for all $t \geq s$. Thus if χ_t is strictly increasing (/decreasing), then $\frac{1}{\chi_s}-1$ is strictly larger (/smaller) than $\rho\mathcal{E}^s$.

As $\mathcal{I}^s =$ holds for all s, we take the derivative w.r.t. s and get

$$\rho\mathcal{I}^s - \iota_{s,s} - \mathcal{U}_{L,s}\dot{H}_s = \rho\mathcal{E}^s - \left(\frac{1}{\chi_s}-1\right)$$

Now supposing that $\dot{\chi}_t > 0$ for all t, we would have $\rho\mathcal{E}^s - \left(\frac{1}{\chi_s}-1\right) < 0$ and thus $\rho\mathcal{I}^s - \iota_{s,s} - \mathcal{U}_{L,s}\dot{H}_s < 0$. If $\dot{\lambda}_t \geq 0$ for all t, we would have $\dot{H}_s \leq 0$ from (41) and $\iota_{s,s} \leq \rho\mathcal{I}^s$, which together imply $\rho\mathcal{I}^s - \iota_{s,s} - \mathcal{U}_{L,s}\dot{H}_s \leq 0$ — a contradiction! Thus if $\dot{\chi}_t > 0$ for all t, we must have $\dot{\lambda}_t < 0$ for all t. Similarly, we can prove that if $\dot{\chi}_t < 0$ for all t, we must have $\dot{\lambda}_t > 0$ for all t.

Step 3: $\dot{\chi}_t < 0$ for all t. Here I further suppress the time subscript when there is no ambiguity. Similar to the proof of Proposition 2, from (38), we have

$$g^\chi = Y'\frac{\chi^2}{\chi+\lambda} - [(1-\alpha)\delta+\rho] - \frac{\lambda}{\chi+\lambda}g^\chi + \frac{\lambda}{\chi+\lambda}\left(1+\chi C\frac{\mathcal{U}_L(\partial\mathcal{U}_L/\partial Z)}{(\partial\mathcal{U}_L/\partial H)}\right)g^\lambda$$

with $g^\chi \equiv \frac{\dot{\chi}}{\chi}, g^\lambda \equiv \frac{\dot{\lambda}}{\lambda}$. Let $K_t \equiv \chi_t C_t \frac{\mathcal{U}_{L,t}(\partial\mathcal{U}_L/\partial Z)_t}{(\partial\mathcal{U}_L/\partial H)_t} > 0$ and $\gamma \equiv \frac{\chi+2\lambda}{\chi+\lambda}g^\chi - \frac{\lambda}{\chi+\lambda}(1+K)g^\lambda$ which has the same sign as g^χ. We have

$$\begin{aligned}
\gamma &= Y'\frac{\chi^2}{\chi+\lambda} - [(1-\alpha)\delta+\rho] \\
\dot{\gamma} &= Y''\dot{Z}\frac{\chi^2}{\chi+\lambda} + Y'\frac{(\chi+2\lambda)\chi^2}{(\chi+\lambda)^2}g^\chi - Y'\frac{\lambda\chi^2}{(\chi+\lambda)^2}g^\lambda \\
&= Y''\dot{Z}\frac{\chi^2}{\chi+\lambda} + Y'\frac{(\chi+2\lambda)\chi^2}{(\chi+\lambda)^2}g^\chi + Y'\frac{\chi^2}{\chi+\lambda}\frac{\gamma-\frac{\chi+2\lambda}{\chi+\lambda}g^\chi}{1+K} \\
&= Y''\dot{Z}\frac{\chi^2}{\chi+\lambda} + Y'\frac{\chi^2}{\chi+\lambda}\frac{\gamma}{1+K} + Y'\frac{(\chi+2\lambda)\chi^2}{(\chi+\lambda)^2}g^\chi\frac{K}{1+K}
\end{aligned}$$

Next I show that $g^\chi < 0$ cannot hold. Suppose for contradiction that it holds, then

$\gamma < 0$ and

$$\dot{\gamma} < Y''\dot{Z}\frac{\chi^2}{\chi+\lambda} + Y'\frac{\chi^2}{\chi+\lambda}\frac{\gamma}{1+K}$$

whose RHS is less than zero too. Thus γ will keep decreasing, which contradicts convergence. Therefore, we conclude that $\dot{\chi} > 0$. Consequently, from step 2, $\dot{\lambda} < 0$. (41) suggests that $\dot{H} > 0$. As H increases over time and $\eta(\lambda)$ decreases in λ and thus increases in time, (40) implies that $\dot{L} > 0$.□

Proof of Proposition 4. The implementability constraint is

$$0 = \int_s^\infty \mathrm{e}^{-\rho(t-s)} \left[(C_t - Y_t) U_{C,t} + \left(L_t - H_0 - F_s^t - g_s \Gamma(Y_t) \right) U_{L,t} \right] \mathrm{d}t, \tag{B9}$$

I construct the Lagrangian as

$$\begin{aligned}\mathcal{L}^{s*} = \max \int_s^\infty \mathrm{e}^{-\rho(t-s)} U(C_t, L_t)\, \mathrm{d}t + \int_s^\infty \mathrm{e}^{-\rho(t-s)} \left\{ \mu_t^{s*} \left[Y(Z_t) - C_t - (\delta+\rho) Z_t \right] + \dot{\mu}_t^{s*} Z_t \right\} \mathrm{d}t + \mu_s^{s*} Z_s \\ + \int_s^\infty \mathrm{e}^{-\rho(t-s)} \zeta_t^{s*} (1 - L_t)\, \mathrm{d}t + \lambda^{s*} \int_s^\infty \mathrm{e}^{-\rho(t-s)} \left[(C_t - Y(Z_t) - T_t) U_{C,t} + \left(L_t - F_s^t - g_s \Gamma(Z_t) \right) U_{L,t} \right] \mathrm{d}t\end{aligned} \tag{B10}$$

The optimality conditions (22-24) become, for any $t \geq s$,

$$0 = U_{C,t} - \lambda^{s*} Y_t U_{CC,t} - \mu_t^{s*} \tag{B11}$$

$$0 = (1 + \lambda^{s*}) U_{L,t} + \lambda^{s*} \left(L_t - H_0 - F_s^t - g_s \Gamma(Y_t) \right) U_{LL,t} - \zeta_t^{s*} \tag{B12}$$

$$0 = \mu_t^{s*} \left[Y_t' - (\delta + \rho) \right] + \dot{\mu}_t^{s*} - \lambda^{s*} \left(Y_t' U_{C,t} + g_s \Gamma_t' U_{L,t} \right) \tag{B13}$$

We need (B13) and (B9) both to hold, which means

$$H_0 + F_s^t + g_s \Gamma(Z_t) = \frac{L^*}{1 + \eta(\lambda^{s*})} \tag{B14}$$

and

$$\mathcal{I}(\lambda^{s*}; L^*) = \mathcal{E}^{s*}$$

with $\mathcal{I}(\lambda^{s*}; L^*) \equiv \frac{\nu(L^*)^{1-\sigma}}{\rho} \frac{1+\lambda^{s*}}{\lambda^{s*}\sigma}$. This latter equation pins down $\lambda^{s*} > \lambda^*$ and the former one determines F_s^t for any $g_s \Gamma(Z_t)$. Then we need to find $g_s \Gamma(Z_t)$ that satisfies (B11) and (B13).

Define an integrating factor $m_t \equiv e^{\int_0^t [Y'_\tau - (\delta+\rho)] d\tau}$ and we have, using (B13)

$$\frac{d}{dt}\left(m_t \frac{\mu_t^{s*}}{\lambda^{s*}}\right) = [Y'_t - (\delta + \rho)]\, m_t \frac{\mu_t^{s*}}{\lambda^{s*}} + m_t \frac{\dot{\mu}_t^{s*}}{\lambda^{s*}}$$
$$= m_t \left(Y'_t U_{C,t} + g_s \Gamma'_t U_{L,t}\right)$$

for all s. Take the difference between a generic s and $s = 0$

$$\frac{d}{dt}\left(m_t \frac{\mu_t^{s*}}{\lambda^{s*}}\right) - \frac{d}{dt}\left(m_t \frac{\mu_t^{*}}{\lambda^{*}}\right) = m_t Y'_t H_s^F F'_t U_{L,t}$$
$$m_t \frac{\mu_t^{s*}}{\lambda^{s*}} - m_s \frac{\mu_s^{s*}}{\lambda^{s*}} = m_t \frac{\mu_t^{*}}{\lambda^{*}} - m_s \frac{\mu_s^{*}}{\lambda^{*}} + g_s \int_s^t m_\tau \Gamma'_\tau U_{L,\tau} d\tau$$

Rewrite (B11) as

$$0 = \frac{U_{C,t} - \mu_t^{s*}}{\lambda^{s*} U_{CC,t}} - Y_t$$

and take the difference between a generic s and $s = 0$ to get

$$\begin{aligned}
0 &= \frac{U_{C,t} - \mu_t^{s*}}{\lambda^{s*} U_{CC,t}} - \frac{U_{C,t} - \mu_t^{*}}{\lambda^{*} U_{CC,t}} = -C_t^*\left(\frac{1}{\lambda^{s*}} - \frac{1}{\lambda^*}\right) + (C_t^*)^2 \left(\frac{\mu_t^{s*}}{\lambda^{s*}} - \frac{\mu_t^{*}}{\lambda^{*}}\right) \\
&= -C_t^*\left(\frac{1}{\lambda^{s*}} - \frac{1}{\lambda^*}\right) + (C_t^*)^2\, m_t^{-1} \left[m_s \left(\frac{\mu_s^{s*}}{\lambda^{s*}} - \frac{\mu_s^{*}}{\lambda^{*}}\right) + H_s^F \int_s^t m_\tau \Gamma'_\tau U_{L,\tau} d\tau\right] \\
0 &= -\frac{m_t}{C_t^*}\left(\frac{1}{\lambda^{s*}} - \frac{1}{\lambda^*}\right) + m_s \left(\frac{\mu_s^{s*}}{\lambda^{s*}} - \frac{\mu_s^{*}}{\lambda^{*}}\right) + g_s \int_s^t m_\tau \Gamma'_\tau U_{L,\tau} d\tau \qquad \text{(B15)}
\end{aligned}$$

Evaluate (B15) at $t = s$

$$0 = -\frac{m_s}{C_s^*}\left(\frac{1}{\lambda^{s*}} - \frac{1}{\lambda^*}\right) + m_s \left(\frac{\mu_s^{s*}}{\lambda^{s*}} - \frac{\mu_s^{*}}{\lambda^{*}}\right)$$

which determines μ_s^{s*} with $\frac{\mu_s^{s*}}{\lambda^{s*}} - \frac{\mu_s^{*}}{\lambda^{*}} < 0$ since $\lambda^{s*} > \lambda^*$.

Take the derivative of (B15) w.r.t. t

$$0 = \frac{m_t}{C_t^*}\left(\frac{1}{\lambda^{s*}} - \frac{1}{\lambda^*}\right)\left(\frac{\dot{C}_t^*}{C_t^*} - \frac{\dot{m}_t}{m_t}\right) + g_s m_t \Gamma'_t U_{L,t}$$

In order for this to hold, we need

$$m_t \Gamma_t' U_{L,t} = k \frac{m_t}{C_t^*} \left(\frac{\dot{C}_t^*}{C_t^*} - \frac{\dot{m}_t}{m_t} \right)$$

holds with a positive constant k. Then

$$g_s = k^{-1} \left(\frac{1}{\lambda^*} - \frac{1}{\lambda^{s*}} \right)$$

We need

$$\begin{aligned}
\Gamma_t' U_{L,t} &= k \frac{1}{C_t^*} \left(\frac{\dot{C}_t^*}{C_t^*} - [Y'(Z_t^*) - (\delta + \rho)] \right) \\
&= -k \frac{\lambda^*}{\lambda^* + \chi_t^*} \left(\chi_t^* Y'(Z_t^*) + \frac{\dot{\chi}_t^*}{\chi_t^*} \right) \qquad \text{(B16)} \\
&= -k \frac{\lambda^*}{\lambda^* + \chi_t^*} \left(\chi_t^* Y'(Z_t^*) + Y'(Z_t^*) \frac{\left(\chi_t^*\right)^2}{\chi_t^* + 2\lambda^*} - \frac{\chi_t^* + \lambda^*}{\chi_t^* + 2\lambda^*} [(1-\alpha)\delta + \rho] \right) \\
&= -k \frac{\lambda^*}{\lambda^* + 2\chi_t^*} [2\chi_t^* Y'(Z_t^*) - [(1-\alpha)\delta + \rho]] \\
\Gamma_t' &= -k \nu^{-1} (L^*)^\sigma \frac{\lambda^*}{\lambda^* + 2\chi_t^*} [2\chi_t^* Y'(Z_t^*) - [(1-\alpha)\delta + \rho]] \qquad \text{(B17)}
\end{aligned}$$

in which we first use (27) and then plug in (B3). (B16) implies that $\Gamma_t' < 0$ — the more output, the less land payment. (B14) then pins down F_s^t.□

C Theory complements

C.1 The role of land tax

C.2 Discussion of Lerner rule

One may wonder why the requirement of demand elasticity less than one behind Lemma 1 seems to go against the usual intuition that the demand elasticity has to be larger than one to admit a monopoly pricing solution. Let us indeed consider such a textbook example (Tirole, 1988, Chapter 1.1): a monopoly quotes a price p under constant marginal cost mc

and demand function $q(p)$ to maximize profits,

$$\max \pi = (p - mc) \underbrace{q(p)}_{\text{demand}} \tag{C1}$$

The optimal price satisfies the familiar Lerner condition

$$\frac{p - mc}{p} = \left(- \frac{\partial \log q(p)}{\partial \log p} \right)^{-1} \tag{C2}$$

The demand elasticity $-\frac{\partial \log q}{\partial \log p}$ has to be larger than one to admit an interior solution. We observe that (C2) and (17) are isomorphic once we swap the price and quantity. In the problem (16), the price declines in supply, but the monopoly does not profit from the pre-owned amount q_0, which acts to erode the profit in a way much like the marginal cost in problem (C1). Thus the requirement for (16) to admit a solution is exactly the opposite: the demand for housing has to be inelastic rather than elastic, which is also empirically relevant. Note that the key difference is not in whether the monopoly sets price or quantity, which maps into each other via the demand function $q(p)$, but rather where a quantity-like term (q_0) or a price-like term (mc) erodes profits.

D Constrained implementations of SB allocation

D.1 No-borrowing implementation

In Section 3.2, the optimal policy is analyzed in complete markets and under the assumption that the government can commit to its future actions. Now we examine if it is possible to implement the same allocation when the government is constrained in one regard. Here we assume that the government can commit, but may not be able to borrow. In the next subsection, we discuss the other case where the government cannot commit but markets are complete.

We evaluate the fiscal position under SB. The time-t income from land lease relative to GDP is, assuming $H_t = H_0$,

$$\frac{D_t^* (L^* - H_0)}{Y_t^*} = \frac{U_L^*}{U_{C,t}^*} \frac{L^* - H_0}{Y_t^*} = U_L^* \eta H_0 \chi_t^*,$$

which is increasing over time. That is because the household values future housing L_t more as their future consumption C_t is higher relative to future GDP Y_t. If the government derives rental income at each time t without selling any land, the fiscal income is back-loaded. In contrast, the time-t expenditure relative to GDP $\frac{\dot{Z}_t^* + \delta Z_t^*}{Y_t^*} = 1 - \chi_t^*$ is front-loaded. This maturity mismatch is an immediate consequence of monotonicity of χ_t^* established in Proposition 2, stated as Corollary D1 below.

Corollary D1. (Fiscal maturity mismatch under SB) *Under the SB allocation, the fiscal expenditure relative to GDP strictly decreases over time. If the government does not sell any land ($H_t = H_0$), the rental income relative to GDP strictly increases over time.*

Nonetheless, we acknowledge that the housing price will incorporate all the future rental income and thus the government could access future rental income via land sale, a point made in Lemma 2 – if the government sells $H_t = L^*$, it can extract all the land value at time 0 and roll over the savings for future expenditure. Further, as the time profiles of sale income and lease income "span" the time profile of SB expenditure, the government can tune the sale/lease ratio to exactly fund its expenditure as each time t. Proposition D1 formalizes this intuition to illustrate that borrowing is not necessary to implement SB.

Proposition D1. *SB allocation could be achieved without borrowing by either the government or the household, if the government can commit to its future actions, by maintaining for all $s \geq 0$*

$$H_s = H_s^{NB} \equiv H_0 + \left[1 - \frac{\int_s^\infty e^{-\rho(t-s)} \left(\frac{1}{\chi_t^*} - 1\right) dt}{\int_0^\infty e^{-\rho t} \left(\frac{1}{\chi_t^*} - 1\right) dt}\right] \eta(\lambda^*) H_0, \tag{D1}$$

The threshold H_s^{NB} increases in s from H_0 to $H_\infty^{NB} = H_0 + \left[1 - \frac{\frac{1}{\chi_\infty^*} - 1}{\rho \int_0^\infty e^{-\rho t}\left(\frac{1}{\chi_t^*} - 1\right) dt}\right] \eta(\lambda^*) H_0$ as s goes from 0 to ∞. That is, the government sells land from H_0 until H_∞^{NB}.

When (D1) holds, the fiscal income exactly offsets the expenditure at any point in time, and no financial transaction is needed. As $s \to \infty$, the amount of land the government owns $L^* - H_\infty^{NB}$ generates exactly enough revenue to replenish infrastructure depreciation. If $\delta = 0$ holds in addition, and thus $\chi_\infty^* = 1$, then the government owns no land in the long run ($H_\infty^{NB} = L^*$). When (D1) holds as inequality with H_s larger than H_s^{NB}, the government saves the fiscal surplus for future investment. In this case, it suffices to have only one debt instrument for the government to use.

D.2 Time-consistent implementation

Lucas and Stokey (1983) unveil the insight that in a closed-economy model with endogenous labor supply but no capital, the optimal fiscal policy to finance exogenous spending with income tax and debt can be implemented in a time-consistent way, provided that the government can issue bonds of any maturity. Debortoli, Nunes and Yared (2021) further qualify this statement. Here we ask, can the optimal policy in our economy be implemented in a time-consistent way? What does that imply for the amount of land H_t that needs to be put in private hands?

Let B_s^τ be a zero-coupon bond with maturity $\tau - s$ issued at time s, which pays one dollar at time τ to the household. If the government reoptimizes at time s, taking as given $\left(Z_s, H_s, \{B_s^\tau\}_{\tau \geq s}\right)$, its implementability constraint (20) takes the following form

$$0 = \int_s^\infty \mathrm{e}^{-\rho(t-s)} \left[\left(C_t - Y_t - B_s^t\right) U_{C,t} + (L_t - H_s)\, U_{L,t}\right] \mathrm{d}t, \tag{D2}$$

which is a discounted sum from time s to ∞. The time-s Lagrangian is

$$\begin{aligned}\mathcal{L}^{s*} = \max \int_0^\infty \mathrm{e}^{-\rho(t-s)} U(C_t, L_t)\, \mathrm{d}t + \int_0^\infty \mathrm{e}^{-\rho(t-s)} \{\mu_t^{s*} [Y_t(Z_t) - C_t - (\delta + \rho) Z_t] + \dot{\mu}_t^{s*} Z_t\}\, \mathrm{d}t + \mu_s^{s*} Z_s \\ + \int_0^\infty \mathrm{e}^{-\rho(t-s)} \zeta_t^{s*} (1 - L_t)\, \mathrm{d}t + \lambda^{s*} \int_0^\infty \mathrm{e}^{-\rho(t-s)} [(C_t - Y_t(Z_t) - T_t)\, U_{C,t} + (L_t - H_s)\, U_{L,t}]\, \mathrm{d}t\end{aligned} \tag{D3}$$

The optimality conditions (22-24) become, for any $t \geq s$,

$$0 = U_{C,t} - \lambda^{s*} \left(Y_t + B_s^t\right) U_{CC,t} - \mu_t^{s*} \tag{D4}$$

$$0 = (1 + \lambda^{s*})\, U_{L,t} + \lambda^{s*} (L_t - H_s)\, U_{LL,t} - \zeta_t^{s*} \tag{D5}$$

$$0 = \mu_t^{s*} [Y_t' - (\delta + \rho)] + \dot{\mu}_t^{s*} - \lambda^{s*} Y_t' U_{C,t} \tag{D6}$$

in which λ^{s*} is the multiplier attached to the implementability constraint (D2), μ_t^{s*} is the multiplier attached to the time-t resource constraint, and ζ_t^{s*} regards the upper bound on time-t land supply. The subscript $s*$ indicates that the optimization is done at time s.

Given the initial condition $\left(Z_s, H_s, \{B_s^\tau\}_{\tau \geq s}\right)$, the time-$s$ planner solves (D2-D6) subject to the resource constraint and land constraint to arrive at the optimal allocation. The resulting multipliers $\left(\lambda^{s*}, \left\{\mu_t^{s*}\right\}_{t \geq s}, \left\{\zeta_t^{s*}\right\}_{t \geq s}\right)$ must all be non-negative. For the time-0 optimal allocation analyzed in Section 3.2 to be carried out by time-s planner, the time-0 planner solves the inverse problem and sets $\left(Z_s, H_s, \{B_s^\tau\}_{\tau\geq s}\right)_{s\geq 0}$ to incentivize future planners. If there exists a solution whose implied multipliers $\left(\lambda^{s*}, \left\{\mu_t^{s*}\right\}_{t\geq s}, \left\{\zeta_t^{s*}\right\}_{t\geq s}\right)_{s\geq 0}$ are all non-negative, then the time-0 optimal allocation can be carried out.

I characterize the set of solutions in Proposition D2, and in doing so, uncover an important property of any time-consistent implementation regarding the amount of land in private hands H_t.

Proposition D2. *SB allocation could be achieved without commitment, if the government can issue bonds in complete markets with a specific maturity structure.*

Specifically, there exists a set of implementation parameterized by $\left\{\lambda^{s*} | \lambda^{s*} \in \left(\frac{1}{\sigma-1}, \infty\right]\right\}_{s>0}$ *such that the land in private hands and the amount of* $(\tau - s)$*-maturity zero-coupon bond the government owes at time s satisfy*

$$H_s = \frac{L^*}{1+\eta\left(\lambda^{s*}\right)} = \frac{\sigma - 1 - \left(\lambda^{s*}\right)^{-1}}{\sigma} L^* \leq \frac{\sigma-1}{\sigma} L^* \tag{D7}$$

$$B_s^\tau = \left(\bar{b}_s^\tau + \tilde{b}_s^\tau\right) C_\tau^*, \ \forall \tau \geq s \tag{D8}$$

with

$$\bar{b}_s^\tau = \frac{C_\tau^* m_\tau^{-1}}{\int_s^\infty \mathrm{e}^{-\rho(t-s)} C_t^* m_t^{-1} \mathrm{d}t} \left(\mathcal{I}\left(\lambda^{s*}; L^*\right) - \mathcal{E}^{s*}\right) \tag{D9}$$

$$\tilde{b}_s^\tau = \left(\frac{C_\tau^* m_\tau^{-1}}{\rho \int_s^\infty \mathrm{e}^{-\rho(t-s)} C_t^* m_t^{-1} \mathrm{d}t} - 1\right)\left(\frac{1}{\lambda^{s*}} - \frac{1}{\lambda^*}\right) \tag{D10}$$

In the expressions above, m_t *is an integrating factor defined as* $m_t \equiv \mathrm{e}^{\int_0^t \left[Y_\tau' - (\delta+\rho)\right] \mathrm{d}\tau}$ *that increases in t,* $\mathcal{I}\left(\lambda^{s*}; L^*\right) \equiv \frac{\nu (L^*)^{1-\sigma}}{\rho} \frac{1+\lambda^{s*}}{\lambda^{s*}\sigma}$ *is the discounted sum of future land income under* $H_s = \frac{L^*}{1+\eta(\lambda^{s*})}$*, and* $\mathcal{E}^{s*} \equiv \int_s^\infty \mathrm{e}^{-\rho(t-s)} \left(\frac{1}{\lambda_t^*} - 1\right) \mathrm{d}t$ *is the discounted sum of future fiscal expenditure. Last, for any* $s > 0$ *and* λ^{s*}*,* B_s^τ / C_τ^* *is either a positive constant across* τ *or strictly monotonic (increasing or decreasing) in* τ*.*

In (D8), the $\bar{b}_s^\tau$ term indicates a baseline position whose sum across the maturity structure equals fiscal surplus (land income minus fiscal expenditure), and the $\tilde{b}_s^\tau$ term is a residual position that averages to zero to fine-tune the maturity structure. The baseline position $\bar{b}_s^\tau$ is positive (/negative) across τ if λ^{s*} is larger (/smaller) than a threshold $\bar{\lambda}^s$, which increases over s from value $\bar{\lambda}^0 = \lambda^*$ at $s = 0$. Further, the $\bar{b}_s^\tau$ terms decreases in absolute value over τ as $C_\tau^* m_\tau^{-1}$ decreases in τ — the further into the future, the smaller bond position relative to consumption. The residual position $\tilde{b}_s^\tau$ is monotonically decreasing (/increasing) if λ^{s*} is less (/greater) than λ^*. If setting $\lambda^{s*} = \lambda^*$ (which implies $H_s = H_0$), we would exactly have $B_s^\tau = C_\tau^* \bar{b}_s^\tau = C_\tau^* \frac{C_\tau^* m_\tau^{-1}}{\int_s^\infty e^{-\rho(t-s)} C_t^* m_t^{-1} dt} \left(\mathcal{I}\left(\lambda^{s*}; L^*\right) - \mathcal{E}^{s*}\right)$ amount of government borrowing for all $\tau \geq s$. In this case, as time $s \to 0$, the bond position approaches zero since $\mathcal{I}\left(\lambda^*; L^*\right) = \mathcal{E}^*$ from Proposition 2.

Moreover, (D7) implies that the land in private hands H_s never exceeds $\frac{\sigma-1}{\sigma} L^*$. To see the intuition, applying Lemma 3 to the time-s decision, the optimal land supply satisfies

$$L^{s*} = \begin{cases} 1, & \lambda^{s*} \leq \frac{1}{\sigma(1-H_s)-1}, \\ \left[1 + \eta\left(\lambda^{s*}\right)\right] H_s < 1, & \lambda^{s*} > \frac{1}{\sigma(1-H_s)-1}, \end{cases}$$

with a decreasing function $\eta\left(\lambda^{s*}\right) = \left(\frac{\lambda^{s*}\sigma}{1+\lambda^{s*}} - 1\right)^{-1}$. As λ^{s*} increases from 0 to ∞, L^{s*} decreases from 1 to $\frac{\sigma}{\sigma-1} H_s$. The logic behind is that, depending on the need for money, the government may supply any amount of land from $\frac{\sigma}{\sigma-1} H_s$ (maximizing profits) to 1 (maximizing housing consumption), but it will never set L^{s*} below the level $\frac{\sigma}{\sigma-1} H_s$. In order for L^{s*} to equal L^*, we must have $H_s \leq \frac{\sigma-1}{\sigma} L^*$; otherwise there is no non-negative solution λ^{s*}. That is, the time-0 planner has to leave $H_s \leq \frac{\sigma-1}{\sigma} L^*$ in private hands to incentivize future planners.

Recalling that any no-borrowing implementation of the SB allocation characterized in Proposition D1 has the property that the government gradually sells land, we establish the following impossibility result.

Corollary D2. *There exists an implementation of the SB allocation that is both time-consistent and involving non-positive* total value *of government debt (i.e. government as net creditor), if and only if*

$$\frac{1}{\chi_\infty^*} - 1 \geq \frac{\nu\left(L^*\right)^{1-\sigma}}{\sigma}. \tag{D11}$$

If δ is small (i.e., $\frac{\alpha\delta}{(1-\alpha)\delta+\rho} \leq \frac{\nu}{\sigma}$), this inequality never holds. Otherwise it holds only when λ^* is small.

Intuitively, if infrastructure never depreciates ($\delta = 0$), in any time-consistent implementation, the government needs no fiscal income at the steady state. Thus it should put all land supply in private hands ($H_\infty^{NB} = L^*$), since it has no debt to repay either. In this case, the SB allocation can never be implemented in a time-consistent way, since $H_\infty^{NB} > \frac{\sigma-1}{\sigma} L^*$.

If δ is large, the government needs lease income at the steady state to replenish infrastructure. When λ^* is large, the steady-state consumption ratio χ^*_∞ is high and investment ratio is low, suggesting a low need of fiscal income. Further, as λ^* is large, the land supply is depressed and the land income is high, the government needs to hold less land to derive a certain amount of income. Taking together, the government chooses H_∞ that exceeds $\frac{\sigma-1}{\sigma}L^*$ when λ^* is large enough, denying (D11).

We conclude this section with a remark that, in cases where (D11) does hold, even though the total value of government debt is non-positive, the government needs the whole maturity structure to implement SB allocation in a time-consistent way, based on Proposition D2.

E Supportive empirical evidence

The theory posits that the combination of discretion and financial constraints makes the government sell and supply more land. To bring it to test, we examine a panel of land supply of Chinese prefecture-level cities that are subject to different extents of financial constraints and discretion. Studying mainland China is an obvious choice here, as it has rich cross-sectional variation compared to Singapore and Hong Kong and better data availability than 19th century US.

I measure financial constraints based on the rate of borrowing of each city, and proxy discretion by an indicator function of whether the top city government official is near the end of their tenure. Following He et al. (2023), an empirical paper that is most related and will be discussed in greater detail, I isolate the variation in borrowing cost that is exogenous to factors determining land supply, using an instrumental variable related to China's 4-trillion stimulus program during the financial crisis. This addresses the threat that a city's borrowing cost can be high exactly because it does not have much land to sell.

E.1 Data and institutional background

Land sale. In mainland China, all urban land is owned by the state and all rural/suburban land owned by rural collectives (local groups of farmers). The state monopolizes the right to requisition rural/suburban land for non-agrarian use. The 1998 Land Management Law authorizes local governments (prefectural- and county-level cities) to sell usufruct right of urban land via short-term leases (shorter than 5 years) or long-term leases (with terms depending on the use of the land). For long-term leases, the longest possible term is 70 years for residential land, 50 years for industrial land (for factories and warehouses), 50 years for land used for public utility (such as schools and hospitals), and 40 years for commercial land (as offices and shopping malls). Land leases fall into the following categories depending on the means of transfer: i) agreement (whose price is not necessarily market-based), ii) tender, auction and listing (at market price), iii) administrative allotment (which is not necessarily priced and is usually used for public purpose), and iv) other means. The first three categories cover the bulk of land transfers and we further restrict attention to the first two categories that represent transfers to the private sector. I use the public data of the universe of land transfers in China with around 3 million observations from 2000m1 to 2022m12. Among the first two categories, residential land and industrial land together account for 82% of the total area supplied and 73% of the total revenue. This sample restriction and coverage aligns with He et al. (2023).

Financial constraints. While the local governments account for more than 70% of total government expenditure, the tax-sharing reform in 1994 that shifted tax revenue from local governments to the central government lowered the share of tax revenue accruing to the local governments to below 50%. The 1995 Budget Law prevents local governments from running deficits or obtaining external financing. In 2009 and 2010, to combat the global financial crisis, the Chinese government undertook a massive fiscal stimulus program of 4 trillion RMB, roughly equivalent to 11% of annual GDP, that involves mostly infrastructure investment. The investment was implemented by local governments, and was largely financed by off-balance-sheet companies (known as local government financing vehicles) on behalf of local governments, which effectively lifted the ban on local governments raising debt. China's shadow banking took off in 2012, accommodating the demand for financing unmet by the traditional banking sector. The tight link of financial development to infrastructure investment seen in post-financial-crisis China can be compared to the growth of US financial markets associated with railroad financing in the National Banking Era (1864-1912).[15] I use the universe of municipal corporate bonds (MCBs), issued by local government financing vehicles and implicitly guaranteed by the corresponding local governments, to construct the MCB yield r_{ct} for city c issued at time t.

[15] Bai, Hsieh and Song (2016) review the details and assess the consequences of China's stimulus program. Chen, He and Liu (2020) study the rise of shadow banking in China in the post-stimulus period, and draw informative analog to the growth of US financial markets in the National Banking Era (1864-1912) to fulfill the unprecedented demand for railroad financing.

Officials' tenure. There are 31 provincial-level regions in mainland China and about 300 prefectural-level cities. The highest office of a provincial (/municipal) area is the provincial (/municipal) party secretary, followed by the governor (/mayor). These officials are appointed to serve 5-year terms that are asynchronous across regions. A commonly held view is that the lower the level of government (e.g. municipal level), the larger the tendency for the party secretary to dominate; the higher the level (e.g. provincial, central), the more division of labor between the party secretary and the head of the government (e.g. governor, premier), with the latter in charge of economic development. I make use of the record of provincial and municipal government officials until the end of 2018, which could be extended further.

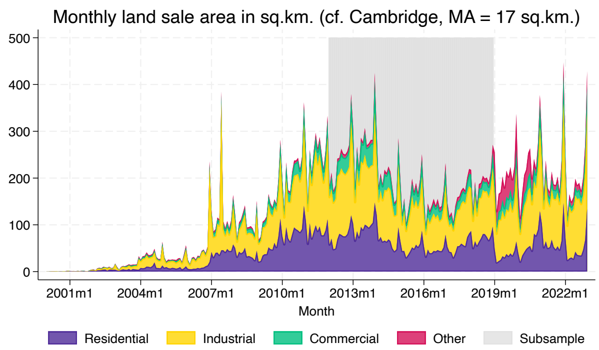
The turnover cycles of Chinese government officials serve two roles in my empirics: i) I use an indicator function $I_{ct}^{\text{LateMuniSec}}$ that equals one if the current municipal party secretary of city i is late in their term at time t, as a proxy of discretion; ii) borrowing from Chen, He and Liu (2020) and He et al. (2023), I use an indicator function $I_{c}^{\text{LateGovernor09}}$ that equals one if the governor of the province city c belongs to was late in their term in 2009, as an instrument for r_{ct}. The idea is that late-term governors are more likely to comply with the central government policy (e.g., stimulus program), as promotion is typically determined by officials immediately above them (known as the "one-level-up" policy). Consequently, the cities under their rule likely engaged in more infrastructure investment in 2009, directed by their governors, and enjoy lower MCB yields later thanks to enhanced fiscal positions. The first stage is strong with the F-statistic above 20 across specifications.

Taking stock, I construct land supply from the land transfer record, measure the financial constraints using MCB yields, and proxy discretion with municipal party secretaries in late terms for all prefecture-level cities from 2012 to 2018. While the first two data sources contain precise transaction dates and the turnover of government officials happens in months, I set a quarterly frequency for empirics which leads to a sample size of about 3000 observations. Quarterly frequency is reasonable considering the time it takes to negotiate land transactions and issue MCBs.[16] Last, I collect city-level data including gross regional product (GRP), population, and fiscal deficit (excluding land revenue) from the City Statistical Yearbooks.

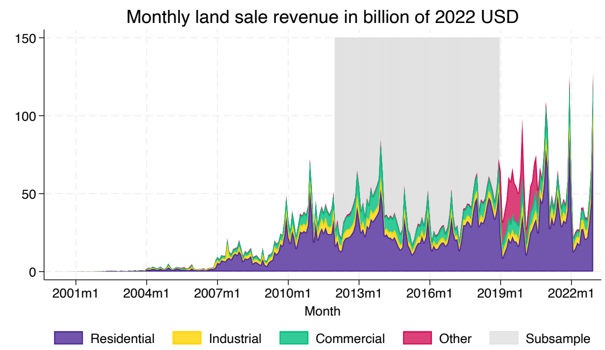
[16] The results are similar for monthly regressions (with about 5000 observations) and annual regressions (with about 1300 observations). The sample selection varies across these regressions since a city may issue MCB in only one month (/quarter) and sell land in another month (/quarter) within a quarter (/year).

Figure E1: Monthly flows of land sale and Municipal Corporate Bond (MCB) issuance

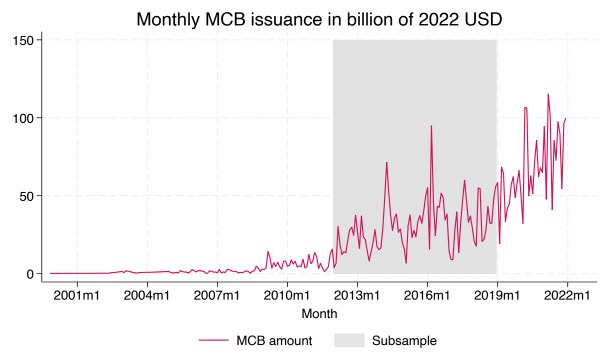
(a) Land sale area



(b) Land sale revenue



(c) MCB issuance



E.2 Empirical findings

The main two-stage least-squares (2SLS) regression I run is

$$\begin{aligned} y_{ct} = {} & \alpha I_{ct}^{\text{LateMuniSec}} + \beta r_{ct} + \gamma I_{ct}^{\text{LateMuniSec}} \cdot r_{ct} \\ & + \sum_{\tau=2012,\ldots,2018} \left(\phi_\tau + \psi_\tau I_{ct}^{\text{LateMuniSec}}\right)' \mathbf{1}_{t=\tau} \left[X_{c,2008}^1, X_{c,t}^2\right] + \text{const} + \delta_t + \epsilon_{ct} \end{aligned} \tag{E1}$$

in which $\left(r_{ct}, I_{ct}^{\text{LateMuniSec}} \cdot r_{ct}\right)$ is instrumented by $\left(I_c^{\text{LateGovernor09}}, I_{ct}^{\text{LateMuniSec}} \cdot I_c^{\text{LateGovernor09}}\right)$. The dependent variable y_{ct} is the land supply by city c at time t normalized by the city population of that year. The regression controls for time-varying effects of two sets of city fundamentals: ex-ante measures $X_{c,2008}^1$ includes GRP per capita, the annual growth rate of GRP, and the fiscal deficit (excluding land revenue) over GRP in 2008; ex-post measures

$X^2_{c,t}$ includes the growth of GRP and land price from 2008 to time t. The theory predicts a positive interaction coefficient γ. The exclusion restriction behind the instrument is that having late-term governors in 2009 affects the land supply decision of cities since 2012 only through the changes in MCB yields. I cluster standard errors at city level.

My regression (E1) extends the following specification in He et al. (2023)

$$y_{ct} = \beta r_{ct} + \sum_{\tau=2012,\ldots,2018} \phi'_\tau \mathbf{1}_{t=\tau}\left[X^1_{c,2008}, X^2_{c,t}\right] + \text{const} + \delta_t + \epsilon_{ct} \tag{E2}$$

which has no interaction terms related to $I^{\text{LateMuniSec}}_{ct}$. They find a positive β coefficient when setting $y_{ct} = \frac{\text{area of residential land supply}_{ct}}{\text{population}_{ct}}$ from 2SLS with r_{ct} instrumented by $I^{\text{LateGovernor09}}_{c}$. That is, a city subject to higher financing cost sells more residential land. This paper further predicts that the slope of land supply to financing cost is steeper if the municipal secretary is in late term.

The first two columns of Table E1 show results from (E1). The coefficient in column (2) means that as the borrowing cost of a city goes up by 1%, it will supply $\beta = 0.198$ ($/\beta + \gamma = 0.353$) sq.m. more residential land per capita in a quarter if its municipal secretary is early (/late) in their term. The interaction coefficient γ is of the correct sign as predicted by the theory but is not statistically significant. I will discuss potential ways to sharpen the result soon. The last two columns estimate (E2) over two subsamples with $I^{\text{LateMuniSec}}_{ct} = 0, 1$ respectively. The estimated β is larger on the subsample with late-term municipal secretary and the magnitude is comparable to columns (1-2). Table E2 displays results using total land supply instead of residential land supply, and the findings are similar.

Table E1: Residential land supply

	Residential land supply/population			
	(1)	(2)	(3)	(4)
MCB yield	0.200	0.198	0.195	0.330
	(0.067)	(0.072)	(0.070)	(0.136)
LateMuniSec=1 × MCB yield	0.125	0.155		
	(0.101)	(0.127)		
Observations	3006	2944	1536	1408
Ex-ante control	Yes	Yes	Yes	Yes
Ex-post control		Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Sample	Full	Full	Early half	Late half
F-stat	26.6	20	36.4	26.1

Standard errors in parentheses

Table E2: Total land supply

	Total land supply/population			
	(1)	(2)	(3)	(4)
MCB yield	0.355	0.380	0.373	0.599
	(0.166)	(0.180)	(0.179)	(0.255)
LateMuniSec=1 × MCB yield	0.230	0.255		
	(0.195)	(0.247)		
Observations	3006	2944	1536	1408
Ex-ante control	Yes	Yes	Yes	Yes
Ex-post control		Yes	Yes	Yes
Time FE	Yes	Yes	Yes	Yes
Sample	Full	Full	Early half	Late half
F-stat	26.6	20	36.4	26.1

Standard errors in parentheses

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